# ECE 344
# Microwave Fundamentals

**Spring 2017**

## Lecture 02

## Transmission Line Theory

# Agenda

– Transmission Line Theory

– The lumped-element circuit model for a transmission line.

– Wave Equations for TL model and their solutions.

– Propagation in lossy & lossless TL.

– Characteristic Impedance.

– Reflection & Transmission Coefficients.

– Standing Wave.

– Input Impedance.

# Transmission-Line Theory

The key difference between circuit theory and transmission line theory is electrical size.

- Lumped circuits: resistors, capacitors, inductors

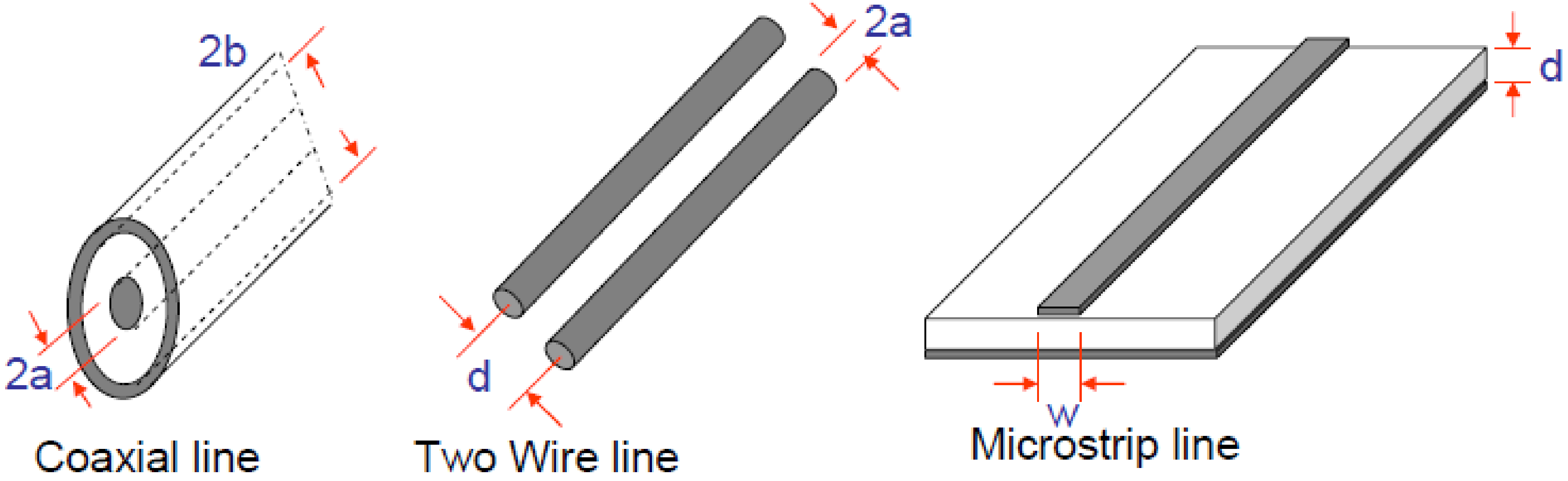
  Neglects time delays (phase change)

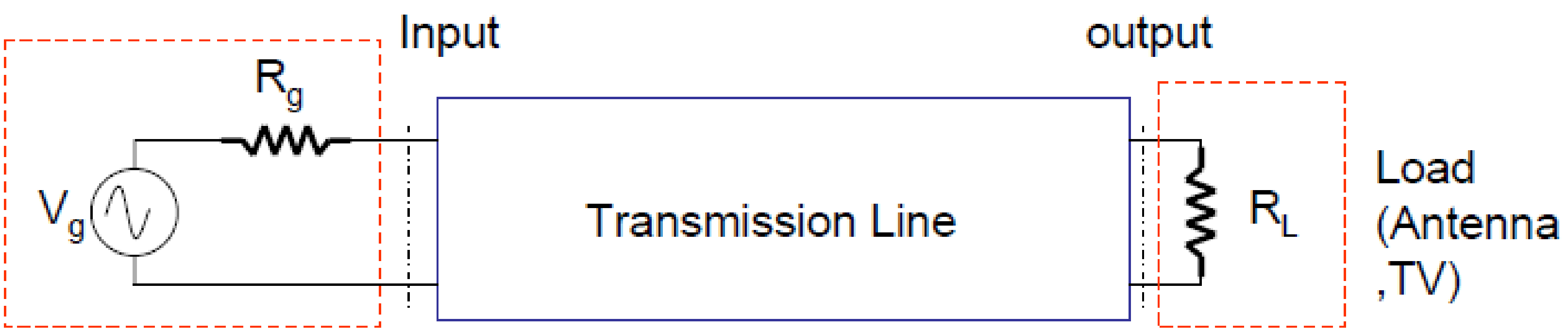
- Distributed circuit elements: transmission lines

  Account for propagation and time delays (phase change)

We need transmission-line theory whenever the length of a line is significant compared to a wavelength.

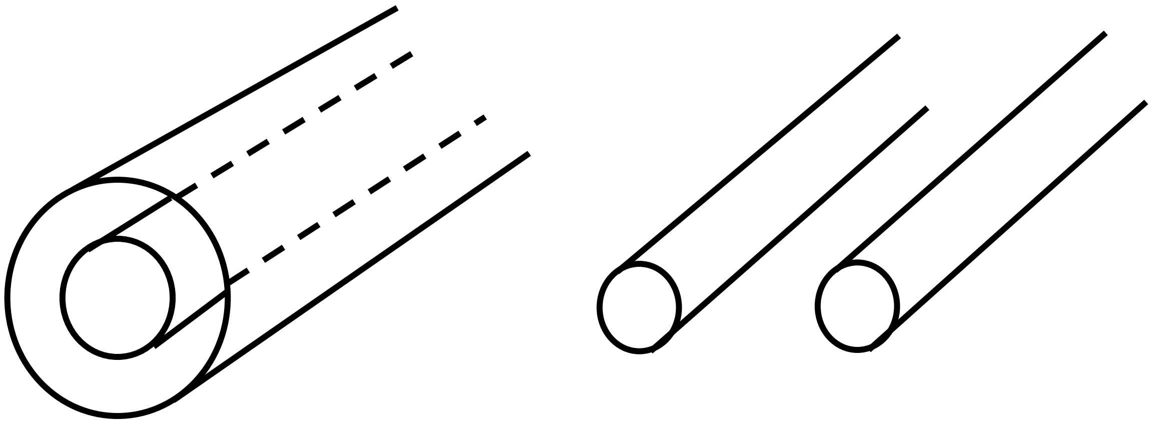
# Transmission Lines



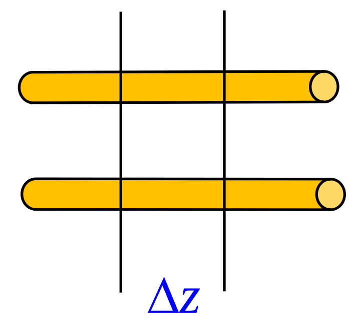



# Transmission Lines

2 conductors



4 per-unit-length parameters:



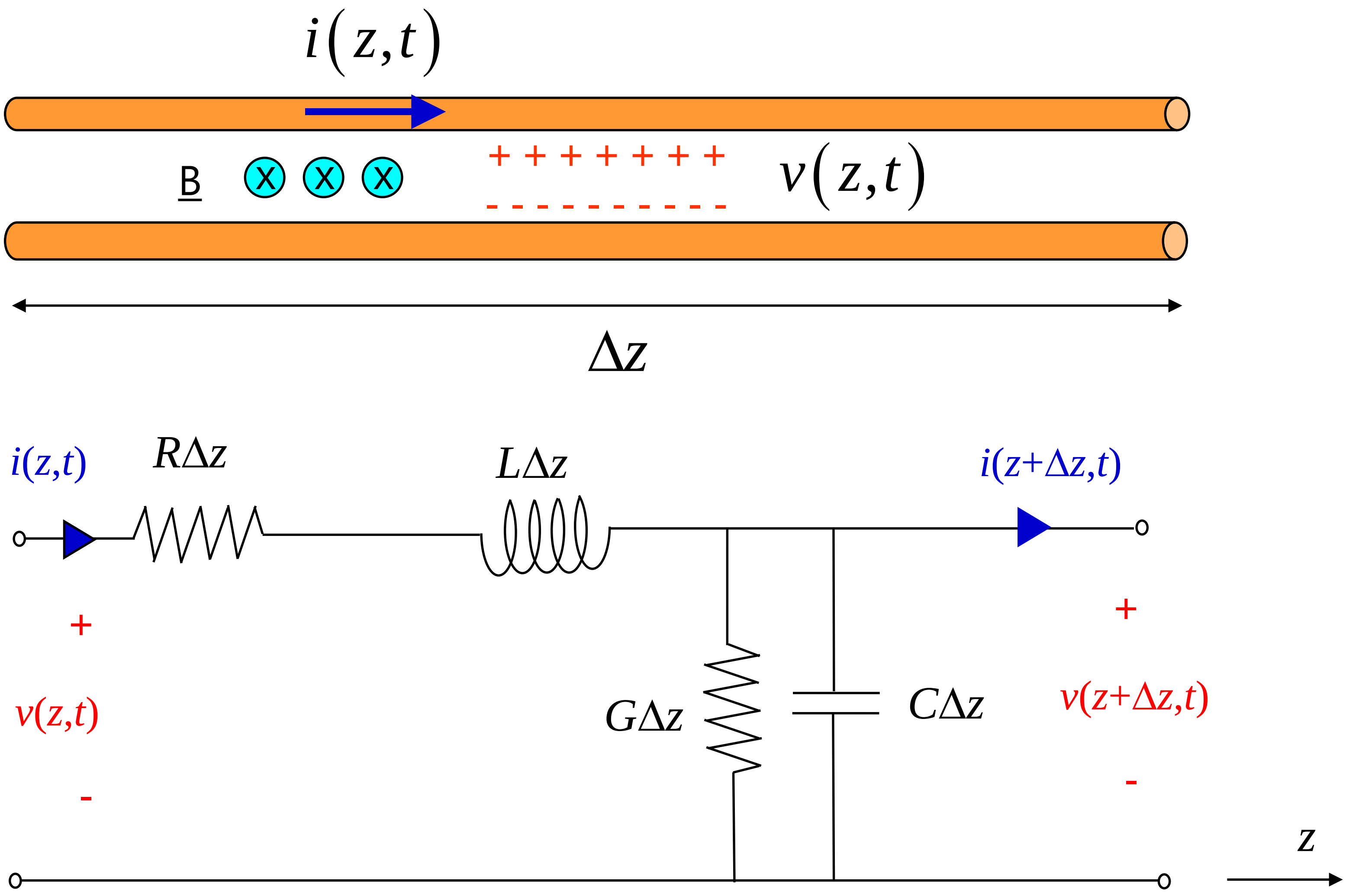

*R* = series resistance per unit length, for both conductors, in Ω/m.

*L* = series inductance per unit length, for both conductors, in H/m.

*G* = shunt conductance per unit length, in S/m or ℧/m.

*C* = shunt capacitance per unit length, in F/m.

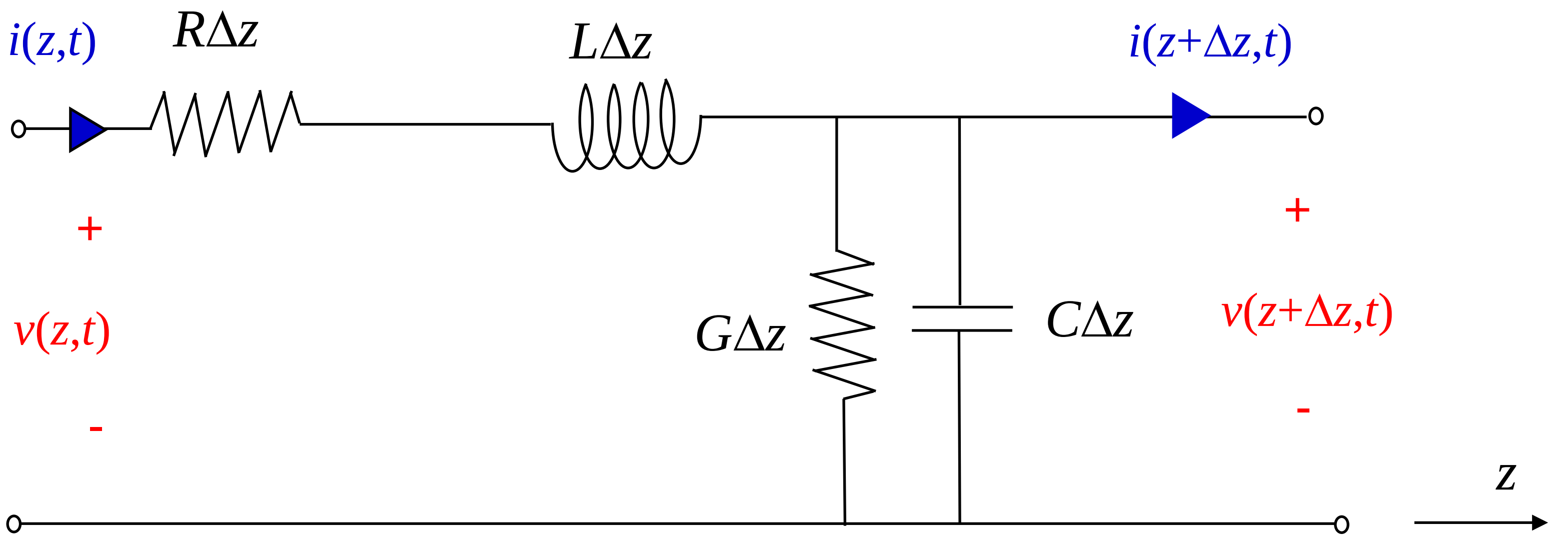
# Transmission Line (cont.)




Note: There are equal and opposite currents on the two conductors.

(We only need to work with the current on the top conductor, since we have chosen to put all of the series elements there.)

# Transmission Line (cont.)




Applying KVL

$$v(z,t) = v(z+\Delta z,t) + i(z,t)R\Delta z + L\Delta z \frac{\partial i(z,t)}{\partial t} \qquad \textbf{(2.1)}$$

Applying KCL

$$i(z,t) = i(z+\Delta z,t) + v(z+\Delta z,t)G\Delta z + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} \qquad \textbf{(2.2)}$$

# TEM Transmission Line (cont.)

Hence

$$\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z}=-Ri(z,t)-L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{i(z+\Delta z,t)-i(z,t)}{\Delta z}=-Gv(z+\Delta z,t)-C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

taking the limit as $\Delta z \to 0$:

$$\frac{\partial v}{\partial z}=-Ri-L\frac{\partial i}{\partial t} \quad \textbf{(2.3)}$$

$$\frac{\partial i}{\partial z}=-Gv-C\frac{\partial v}{\partial t} \quad \textbf{(2.4)}$$

"Telegrapher's Equations"

**These are the time domain form of the transmission line equations,**

# TEM Transmission Line (cont.)

From instantaneous to phasor form $\dfrac{\partial}{\partial t} \rightarrow j\omega$

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z) \qquad \textbf{(2.5)}$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z) \qquad \textbf{(2.6)}$$

To combine these, take the derivative of the first one with respect to $z$:

$$\frac{\partial^2 V(z)}{\partial z^2} = -(R + j\omega L)\frac{\partial I(z)}{\partial z}$$

$$\frac{\partial^2 V(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)V(z)$$

The same differential equation also holds for $i$.

# TEM Transmission Line (cont.)

This yields to give wave equations for *V(z)* and *I (z)*:

$$\frac{\partial^2 V(z)}{\partial z^2} - \gamma^2 V(z) = 0 \qquad \textbf{(2.7)}$$

$$\frac{\partial^2 I(z)}{\partial z^2} - \gamma^2 I(z) = 0 \qquad \textbf{(2.8)}$$

**Where** $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

is the complex propagation constant, which is a function of frequency.

**Equations (2.7) and (2.8) are 2nd order differential equations**

Solution: $V(z) = Ae^{-\gamma z} + Be^{+\gamma z}$

# TEM Transmission Line (cont.)

Traveling wave solutions to (2.7) and (2.8) can be found as

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad \textbf{(2.9)}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \quad \textbf{(2.10)}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Denote: $\gamma = \alpha + j\beta$

$\gamma =$ propagation constant $[1/\text{m}]$

$\beta =$ phase constant $[\text{rad/m}]$

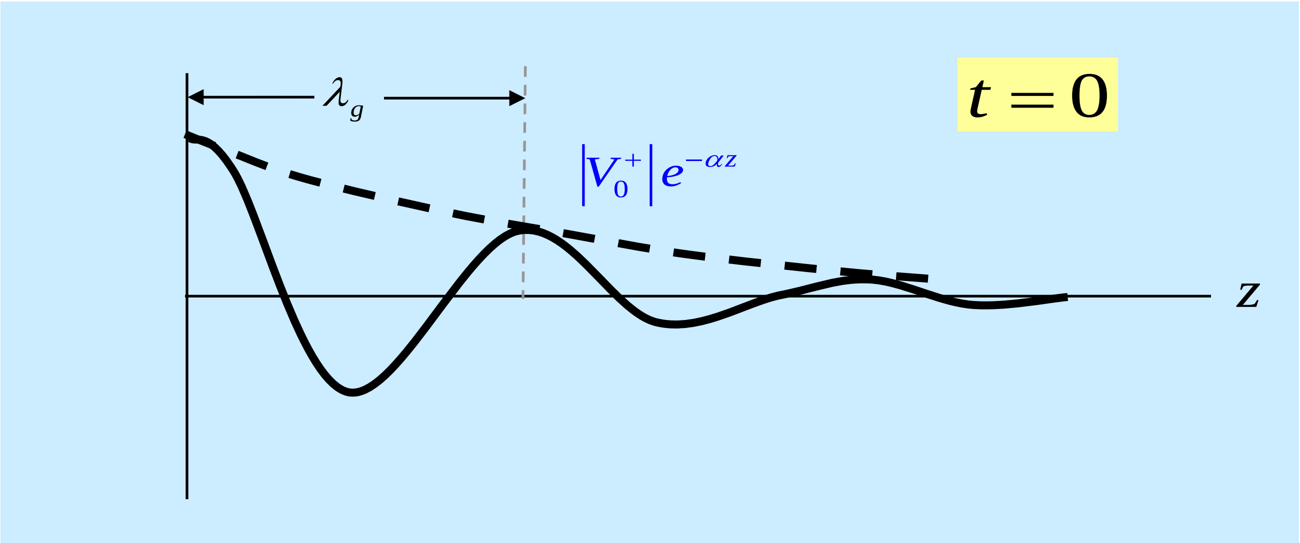
$\alpha =$ attenuation contant $[\text{np/m}]$

# TEM Transmission Line (cont.)

Forward travelling wave (a wave traveling in the positive $z$ direction):

$$V^{+}(z) = V_0^{+}\, e^{-\gamma z} = V_0^{+}\, e^{-\alpha z}\, e^{-j\beta z}$$

$$\begin{aligned} v^{+}(z,t) &= \mathrm{Re}\left\{\left(V_0^{+}\, e^{-\alpha z}\, e^{-j\beta z}\right) e^{j\omega t}\right\} \\ &= \mathrm{Re}\left\{\left(\left(\left|V_0^{+}\right| e^{j\phi}\right) e^{-\alpha z}\, e^{-j\beta z}\right) e^{j\omega t}\right\} \\ &= \left|V_0^{+}\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi\right) \end{aligned}$$




The wave "repeats" when:

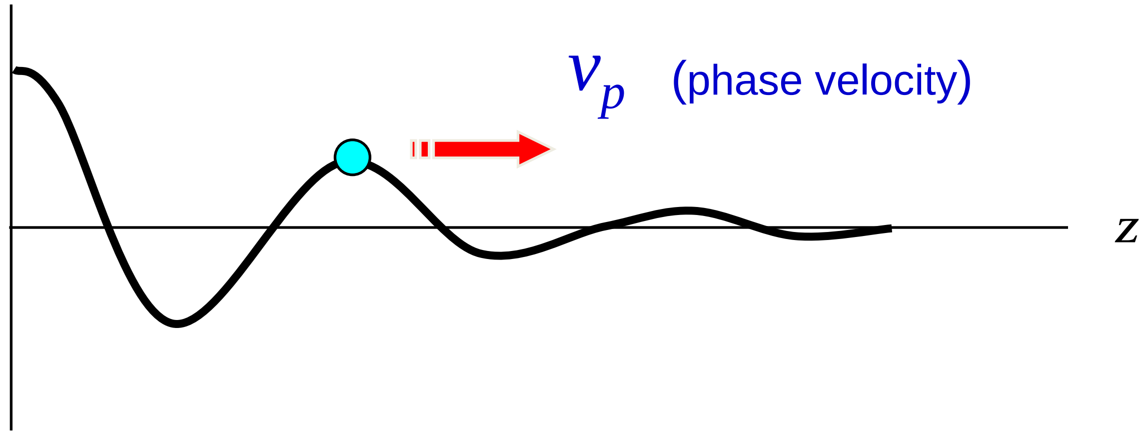
$$\beta \lambda_g = 2\pi$$

Hence:

$$\beta = \frac{2\pi}{\lambda_g}$$

# Phase Velocity

Let's track the velocity of a fixed point on the wave (a point of constant phase), e.g., the crest of the wave.




$$v^{+}(z,t) = \left|V_0^{+}\right| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

# Phase Velocity (cont.)

For any point on the wave the term $\omega t$-$\beta z$ should stay constant,

Set $\omega t - \beta z = \text{constant}$

$$\omega - \beta \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

Hence $$v_p = \frac{\omega}{\beta}$$

# General Case (Waves in Both Directions)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$= \left|V_0^+\right| e^{j\phi^+} e^{-\alpha z} e^{-j\beta z} + \left|V_0^-\right| e^{j\phi^-} e^{+\alpha z} e^{+j\beta z}$$

Wave in $+z$ direction

Wave in -$z$ direction

In the time domain:

$$v(z,t) = \mathrm{Re}\left\{V(z) e^{j\omega t}\right\}$$

$$= \left|V_0^+\right| e^{-\alpha z} \cos\left(\omega t - \beta z + \phi^+\right)$$

$$+ \left|V_0^-\right| e^{+\alpha z} \cos\left(\omega t + \beta z + \phi^-\right)$$

# Characteristic Impedance $Z_0$ (cont.)

Applying (2.5) to the voltage of (2.9) gives the current on the line:

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z) \qquad \textbf{(2.5)}$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \qquad \textbf{(2.9)}$$

$$-\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} = -(R + j\omega L)I(z)$$

so

$$I(z) = \frac{\gamma}{(R + j\omega L)}(V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z})$$

**Comparison with (2.10) shows that a characteristic impedance, $Z_0$, can be defined as**

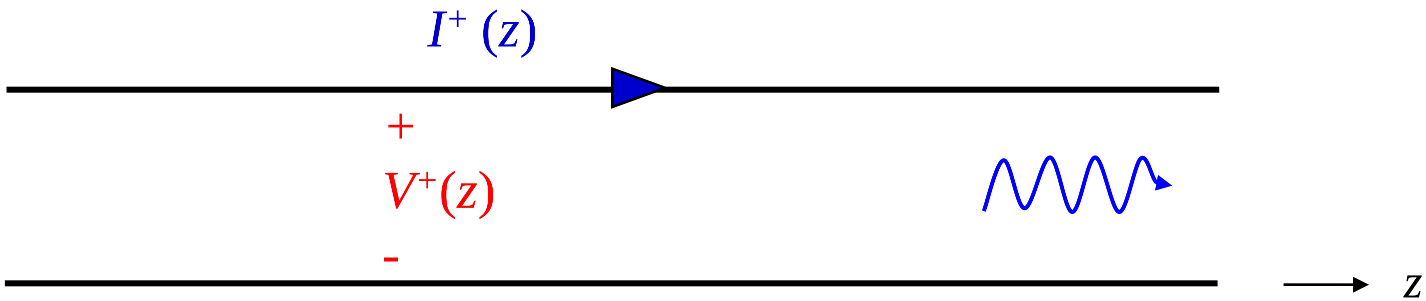
$$Z_0 = \frac{(R + j\omega L)}{\gamma} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \qquad \textbf{(2.11)}$$

# Characteristic Impedance $Z_0$ (cont.)

$$\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z} \qquad \textbf{(2.12)}$$

# Characteristic Impedance $Z_0$




Assumption: A wave is traveling in the positive $z$ direction.

$$Z_0 \equiv \frac{V^+(z)}{I^+(z)}$$

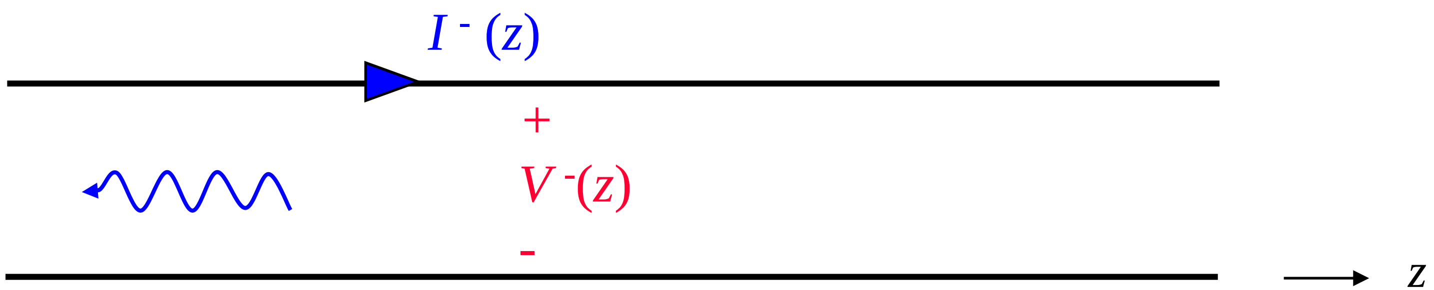
$$V^+(z) = V_0^+ e^{-\gamma z}$$

$$I^+(z) = I_0^+ e^{-\gamma z}$$

so $$Z_0 = \frac{V_0^+}{I_0^+}$$

(Note: $Z_0$ is a number, not a function of $z$.)

# Backward-Traveling Wave

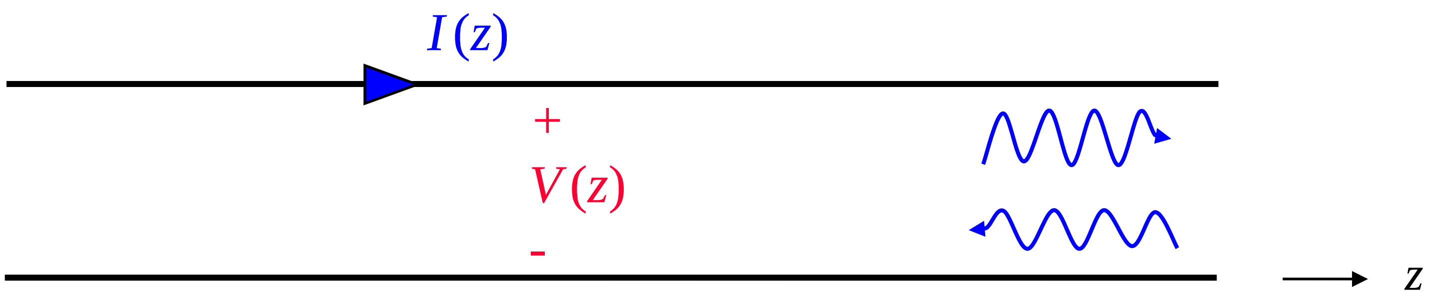



A wave is traveling in the negative $z$ direction.

$$\frac{V^-(z)}{-I^-(z)} = Z_0 \qquad \text{so} \qquad \frac{V^-(z)}{I^-(z)} = -Z_0$$

Note: The reference directions for voltage and current are chosen the same as for the forward wave.

# General Case



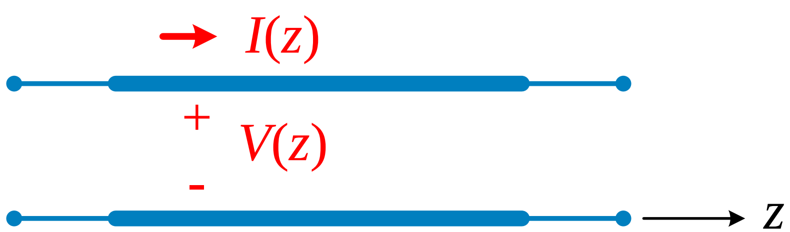

Most general case: A general superposition of forward and backward traveling waves:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{1}{Z_0}\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right]$$

Note: The reference directions for voltage and current are the same for forward and backward waves.

# Summary of Basic TL formulas




$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Guided wavelength

$$\lambda_g = \frac{2\pi}{\beta}\,[\mathrm{m}]$$

Phase velocity

$$v_p = \frac{\omega}{\beta}\,[\mathrm{m/s}]$$

Attenuation in $\mathrm{dB/m} = 8.686\alpha$

Attenuation in $\mathrm{dB/m} = -20\log_{10}\left(e^{-\alpha(1)}\right) = 8.686\alpha$

Note:
$\log_{10}(x) = 0.4343\ln(x)$

# Lossless Case

In many practical cases, however, the loss of the line is very small and so can be neglected, resulting in a simplification of the results.

$$R = 0, \; G = 0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= j\omega\sqrt{LC}$$

so

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \Rightarrow Z_0 = \sqrt{\frac{L}{C}}$$

(real and independent of freq.)

$$v_p = \frac{1}{\sqrt{LC}}$$

(independent of freq.)

# Lossless Case

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

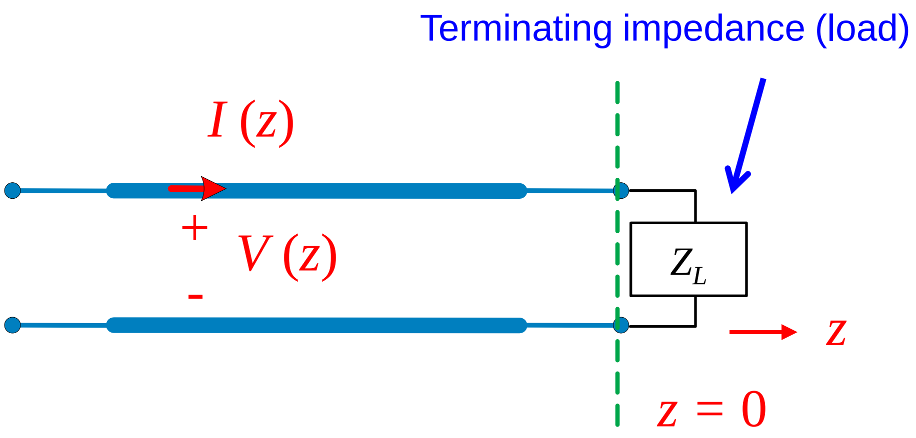
$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$$

# Terminated Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

Amplitude of voltage wave propagating in positive $z$ direction at $z = 0$.

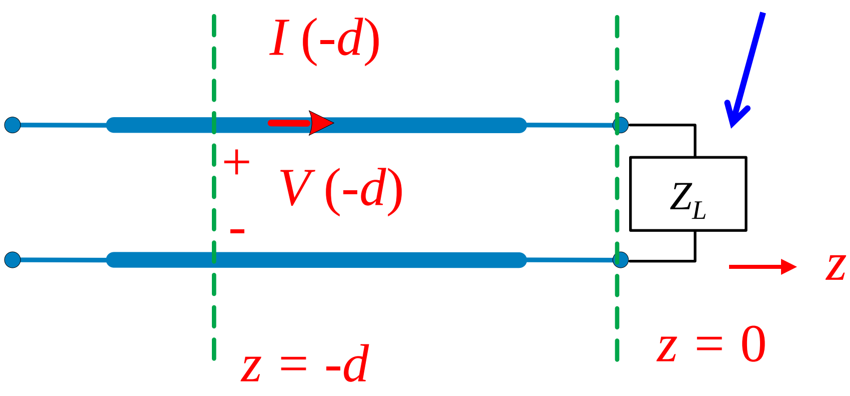
Amplitude of voltage wave propagating in negative $z$ direction at $z = 0$.




**Where do we assign $z = 0$ ?**
The usual choice is at the load.

# Terminated Transmission Line (cont.)

What is $V(-d)$ ?




$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$V(-d) = V_0^+ e^{\gamma d} + V_0^- e^{-\gamma d}$$

Propagating forwards

Propagating backwards

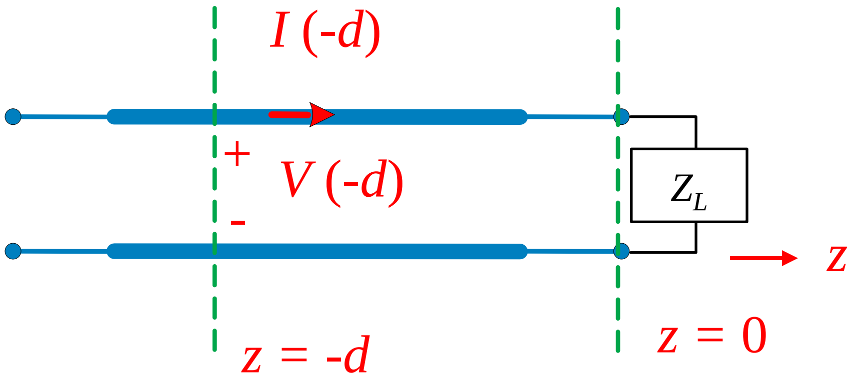
The current at $z = -d$ is

$$I(-d) = \frac{V_0^+}{Z_0} e^{\gamma d} - \frac{V_0^-}{Z_0} e^{-\gamma d}$$

$d \equiv$ distance away from load

(This does not necessarily have to be the length of the entire line.)

# Terminated Transmission Line (cont.)




$$V(-d) = V_0^+ e^{\gamma d} + V_0^- e^{-\gamma d} = V_0^+ e^{\gamma d}\left(1 + \frac{V_0^-}{V_0^+} e^{-2\gamma d}\right)$$

or

$$V(-d) = V_0^+ e^{\gamma d}\left(1 + \Gamma_L e^{-2\gamma d}\right)$$

$\Gamma_L \equiv$ Load reflection coefficient

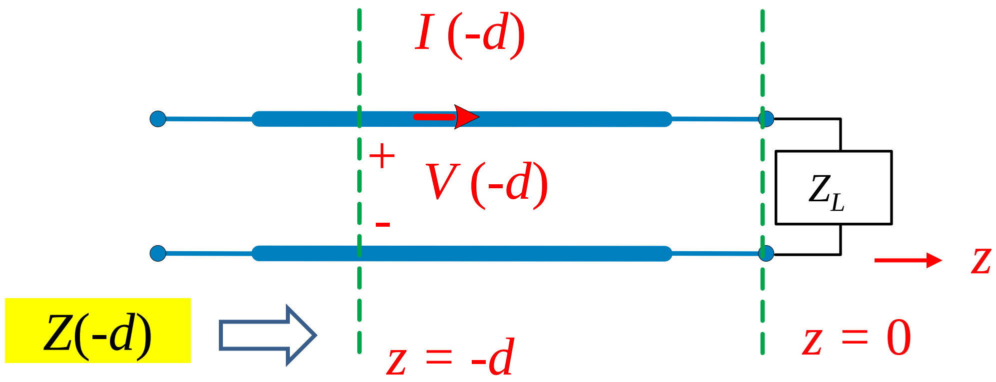
$$\Gamma_L \equiv \frac{V_0^-}{V_0^+}$$

Similarly,

$$I(-d) = \frac{V_0^+}{Z_0} e^{\gamma d}\left(1 - \Gamma_L e^{-2\gamma d}\right)$$

# Terminated Transmission Line (cont.)

$Z(-d)$ = impedance seen "looking" towards load at $z = -d$.




Note:
If we are at the beginning of the line, we will call this "input impedance".

$$V(-d) = V_0^+ e^{\gamma d}\left(1+\Gamma_L e^{-2\gamma d}\right)$$

$$I(-d) = \frac{V_0^+}{Z_0} e^{\gamma d}\left(1-\Gamma_L e^{-2\gamma d}\right)$$

$$\Rightarrow \quad Z(-d) = \frac{V(-d)}{I(-d)} = Z_0\left(\frac{1+\Gamma_L e^{-2\gamma d}}{1-\Gamma_L e^{-2\gamma d}}\right)$$

# Terminated Transmission Line (cont.)

At the load ($d = 0$):

$$Z(0) = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) \equiv Z_L$$

$$\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Recall $\quad Z(-d) = Z_0\left(\dfrac{1+\Gamma_L e^{-2\gamma d}}{1-\Gamma_L e^{-2\gamma d}}\right)$

Thus,

$$Z(-d) = Z_0\left(\frac{1+\left(\dfrac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma d}}{1-\left(\dfrac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma d}}\right)$$

# Terminated Transmission Line (cont.)

Simplifying, we have:

$$Z(-d) = Z_0 \left( \frac{1 + \left( \frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2\gamma d}}{1 - \left( \frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2\gamma d}} \right) = Z_0 \left( \frac{(Z_L + Z_0) + (Z_L - Z_0) e^{-2\gamma d}}{(Z_L + Z_0) - (Z_L - Z_0) e^{-2\gamma d}} \right)$$

$$= Z_0 \left( \frac{(Z_L + Z_0) e^{+\gamma d} + (Z_L - Z_0) e^{-\gamma d}}{(Z_L + Z_0) e^{+\gamma d} - (Z_L - Z_0) e^{-\gamma d}} \right)$$

$$= Z_0 \left( \frac{Z_L \cosh(\gamma d) + Z_0 \sinh(\gamma d)}{Z_0 \cosh(\gamma d) + Z_L \sinh(\gamma d)} \right)$$

Hence, we have

$$Z(-d) = Z_0 \left( \frac{Z_L + Z_0 \tanh(\gamma d)}{Z_0 + Z_L \tanh(\gamma d)} \right)$$

# Terminated Lossless Transmission Line

Lossless: $\gamma = \alpha + j\beta = j\beta$

$$V(-d) = V_0^+ e^{j\beta d}\left(1+\Gamma_L e^{-2j\beta d}\right)$$

$$I(-d) = \frac{V_0^+}{Z_0} e^{j\beta d}\left(1-\Gamma_L e^{-2j\beta d}\right)$$

$$Z(-d) = Z_0\left(\frac{1+\Gamma_L e^{-2j\beta d}}{1-\Gamma_L e^{-2j\beta d}}\right)$$

$$Z(-d) = Z_0\left(\frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}\right)$$

Note: $\tanh(\gamma d) = \tanh(j\beta d) = j\tan(\beta d)$

Impedance is periodic with period $\lambda_g/2$:

The tan function repeats when

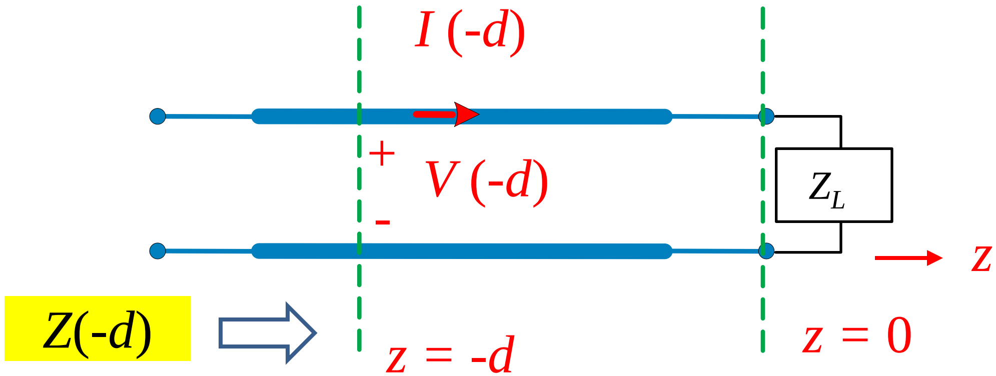
$$\beta(d_2 - d_1) = \pi$$

$$\frac{2\pi}{\lambda_g}(d_2 - d_1) = \pi$$

$$\Rightarrow d_2 - d_1 = \lambda_g / 2$$

# Summary for Lossless Transmission Line

Note: For the remainder of our transmission line discussion we will assume that the transmission line is lossless.




$$Z(-d) = Z_0\left(\frac{1+\Gamma_L e^{-2j\beta d}}{1-\Gamma_L e^{-2j\beta d}}\right)$$

$$= Z_0\left(\frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}\right)$$

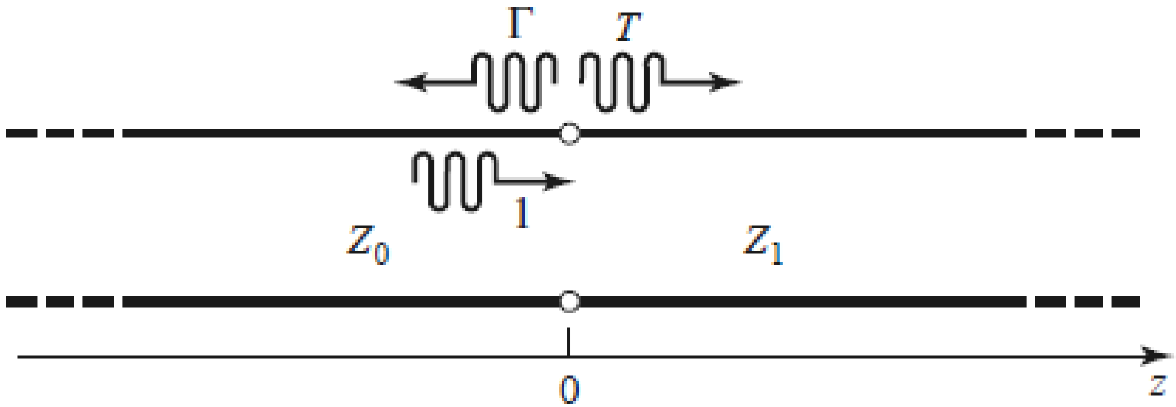
$$\Gamma(\ell) = \frac{V_o^- e^{-j\beta\ell}}{V_o^+ e^{j\beta\ell}} = \Gamma(0)e^{-2j\beta\ell}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\lambda_g = \frac{2\pi}{\beta}$$

$$v_p = \frac{\omega}{\beta}$$

Reflection and transmission at the junction of two transmission lines with different characteristic impedances.




reflection coefficient is given by $$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$ RL = −20 log |Γ| dB,

the voltage for $z < 0$ is $$V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{+j\beta z}), \qquad z < 0$$

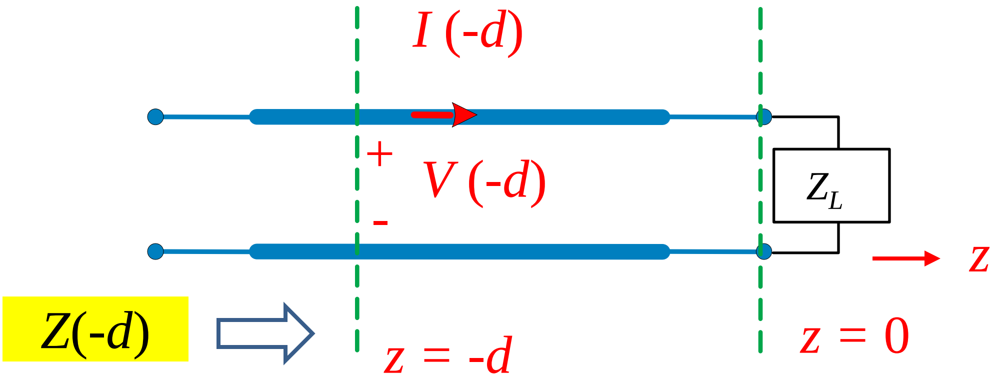
the voltage for $z > 0$ is $$V(z) = V_0^+ T e^{-j\beta z}, \qquad z > 0$$

Equating these voltages at $z = 0$ gives the *transmission coefficient*, $T$ , as

$$T = 1 + \Gamma = \frac{2Z_1}{Z_1 + Z_0}$$

The transmission coefficient between two points in a circuit is often expressed in dB as the insertion loss, IL, $IL = -20 \log |T| \quad \text{dB}$

# Matched Load ($Z_L = Z_0$)




$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

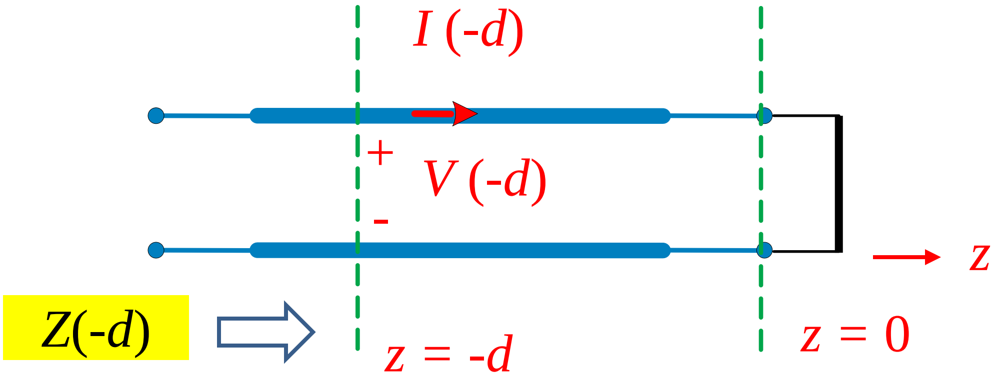
No reflection from the load

$$Z(-d) = Z_0 \left( \frac{1 + \Gamma_L e^{-2j\beta d}}{1 - \Gamma_L e^{-2j\beta d}} \right) \qquad \Rightarrow Z(-d) = Z_0 \text{ for any } z$$

# Short-Circuit Load ($Z_L$=0)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = -1$$




$$Z(-d) = Z_0 \left( \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} \right)$$

$$Z(-d) = jZ_0 \tan(\beta d)$$

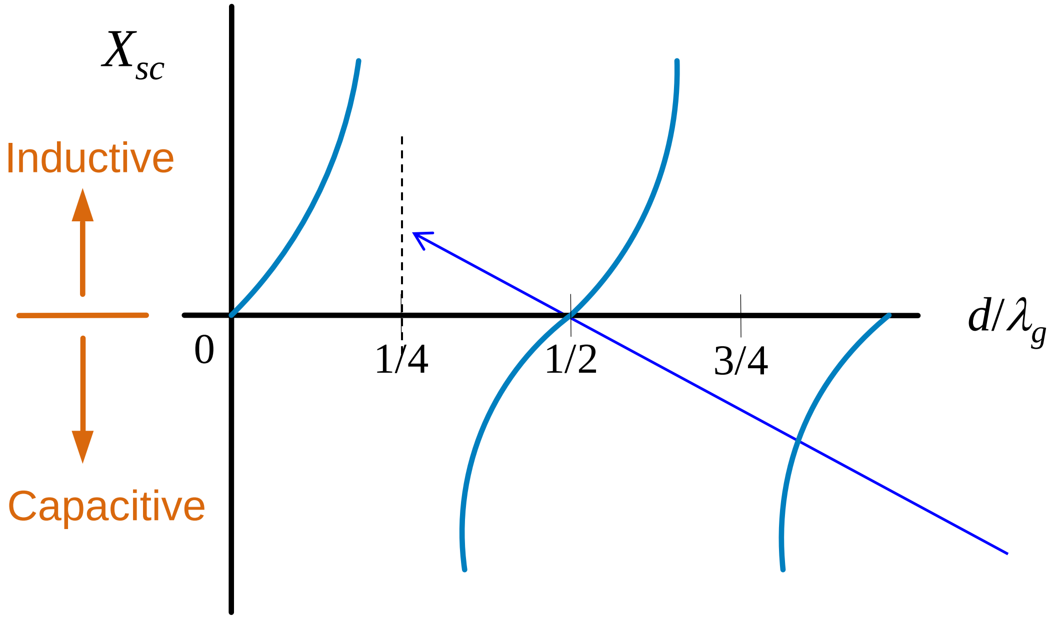
Always imaginary!

# Short-Circuit Load ($Z_L$=0)

$$Z(-d) = jZ_0 \tan(\beta d)$$

$$Z(-d) = jX_{sc}$$

$$X_{sc} = Z_0 \tan(\beta d)$$




Note: $\beta d = 2\pi \dfrac{d}{\lambda_g}$

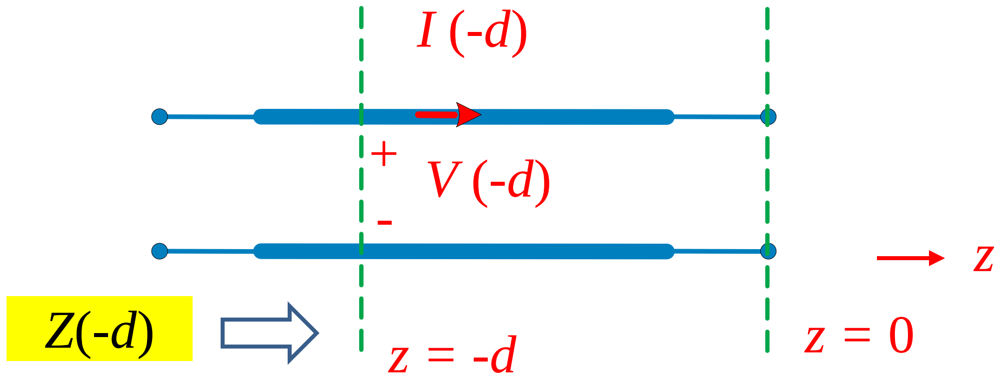
S.C. can become an O.C. with a $\lambda_g/4$ transmission line.

# Open-Circuit Load ($Z_L = \infty$)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{\infty - Z_0}{\infty + Z_0}$$

$$\Gamma_L = +1$$




$$Z(-d) = Z_0\left(\frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}\right)$$

or

$$Z(-d) = Z_0\left(\frac{1 + j(Z_0/Z_L)\tan(\beta d)}{(Z_0/Z_L) + j\tan(\beta d)}\right)$$

$$Z(-d) = -jZ_0\cot(\beta d)$$

Always imaginary!

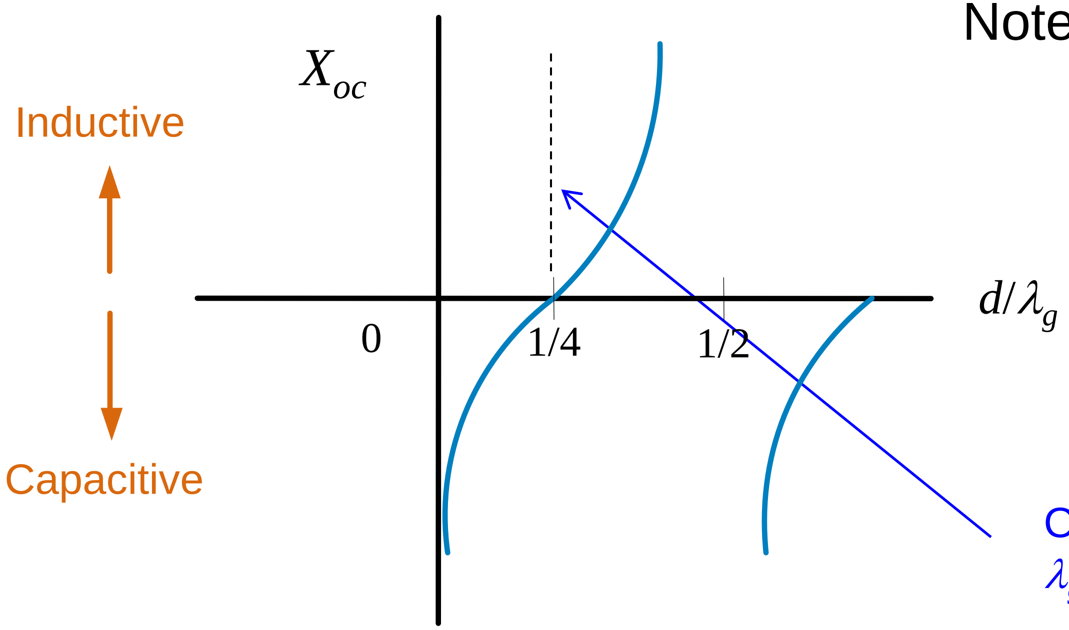
# Open-Circuit Load ($Z_L = \infty$)

$$Z(-d) = -jZ_0 \cot(\beta d)$$

$$Z(-d) = jX_{oc}$$

$$X_{oc} = -Z_0 \cot(\beta d)$$

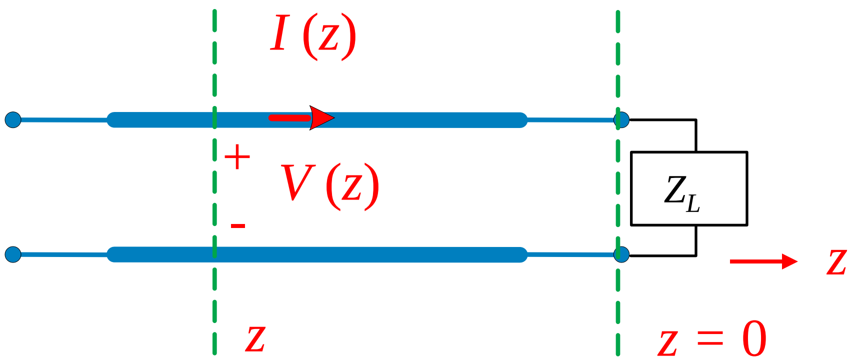
Note: $\beta d = 2\pi \dfrac{d}{\lambda_g}$




O.C. can become a S.C. with a $\lambda_g/4$ transmission line.

# Voltage Standing Wave

$$V(z) = V_0^+ e^{-j\beta z}\left(1+\Gamma_L e^{+2j\beta z}\right)$$
$$= V_0^+ e^{-j\beta z}\left(1+|\Gamma_L| e^{j\phi_L} e^{+2j\beta z}\right)$$



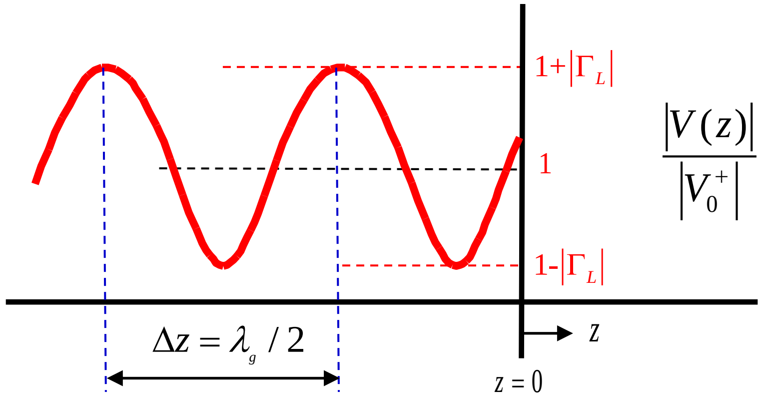

$$|V(z)| = |V_0^+|\left|1+|\Gamma_L| e^{j\phi_L} e^{+j2\beta z}\right|$$

Maximum Occurs When

$$\phi_L + 2\beta z = 0, -2\pi, \ldots\ldots, -2n\pi$$

Minimum Occurs When

$$\phi_L + 2\beta z = -\pi, -3\pi, \ldots, -(2n+1)\pi$$

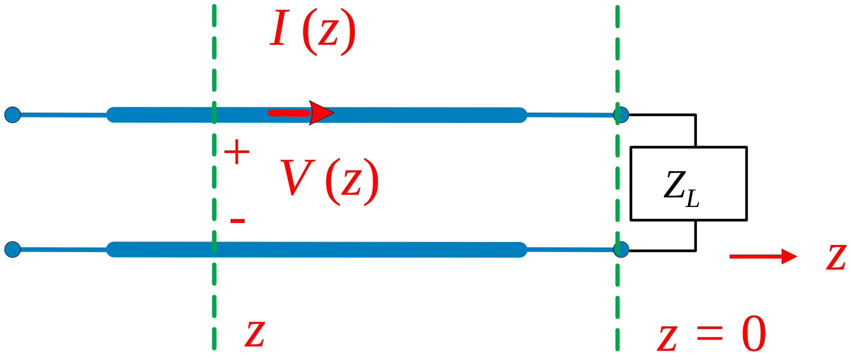



$$V_{max} = |V_0^+|\left(1+|\Gamma_L|\right)$$
$$V_{min} = |V_0^+|\left(1-|\Gamma_L|\right)$$

$$2\beta\Delta_z = 2\pi \quad \Rightarrow \quad \Delta_z = \lambda_g / 2$$

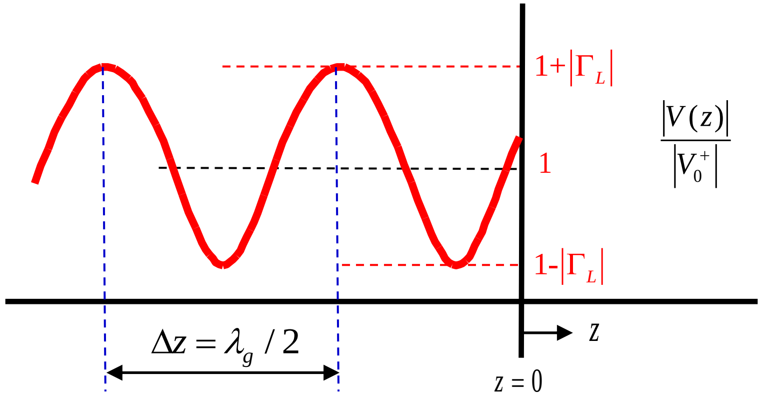
# Voltage Standing Wave Ratio

$$\text{Voltage Standing Wave Ratio (VSWR)} \equiv \frac{V_{\max}}{V_{\min}}$$



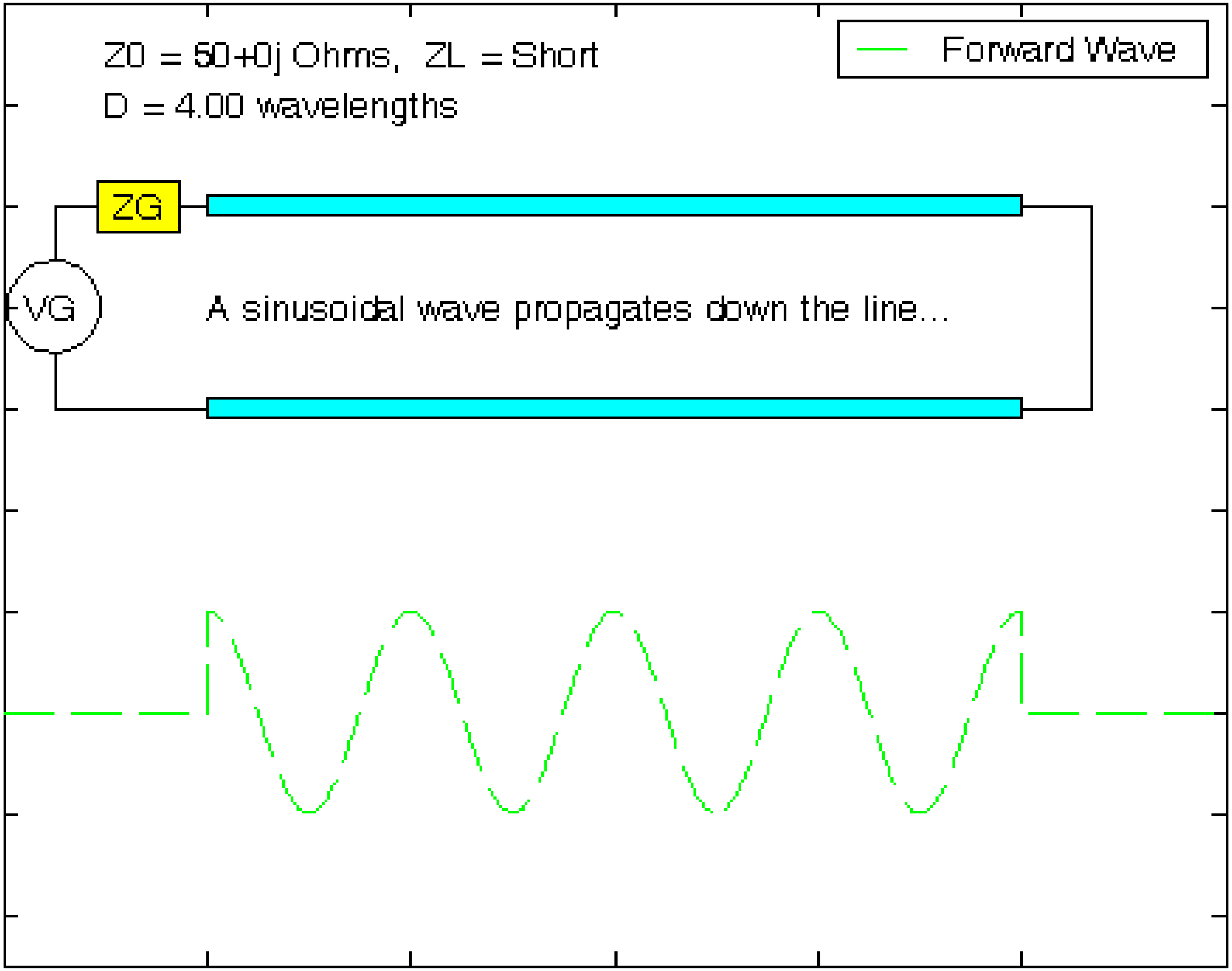

$$V_{\max} = \left|V_0^+\right|\left(1+\left|\Gamma_L\right|\right)$$

$$V_{\min} = \left|V_0^+\right|\left(1-\left|\Gamma_L\right|\right)$$

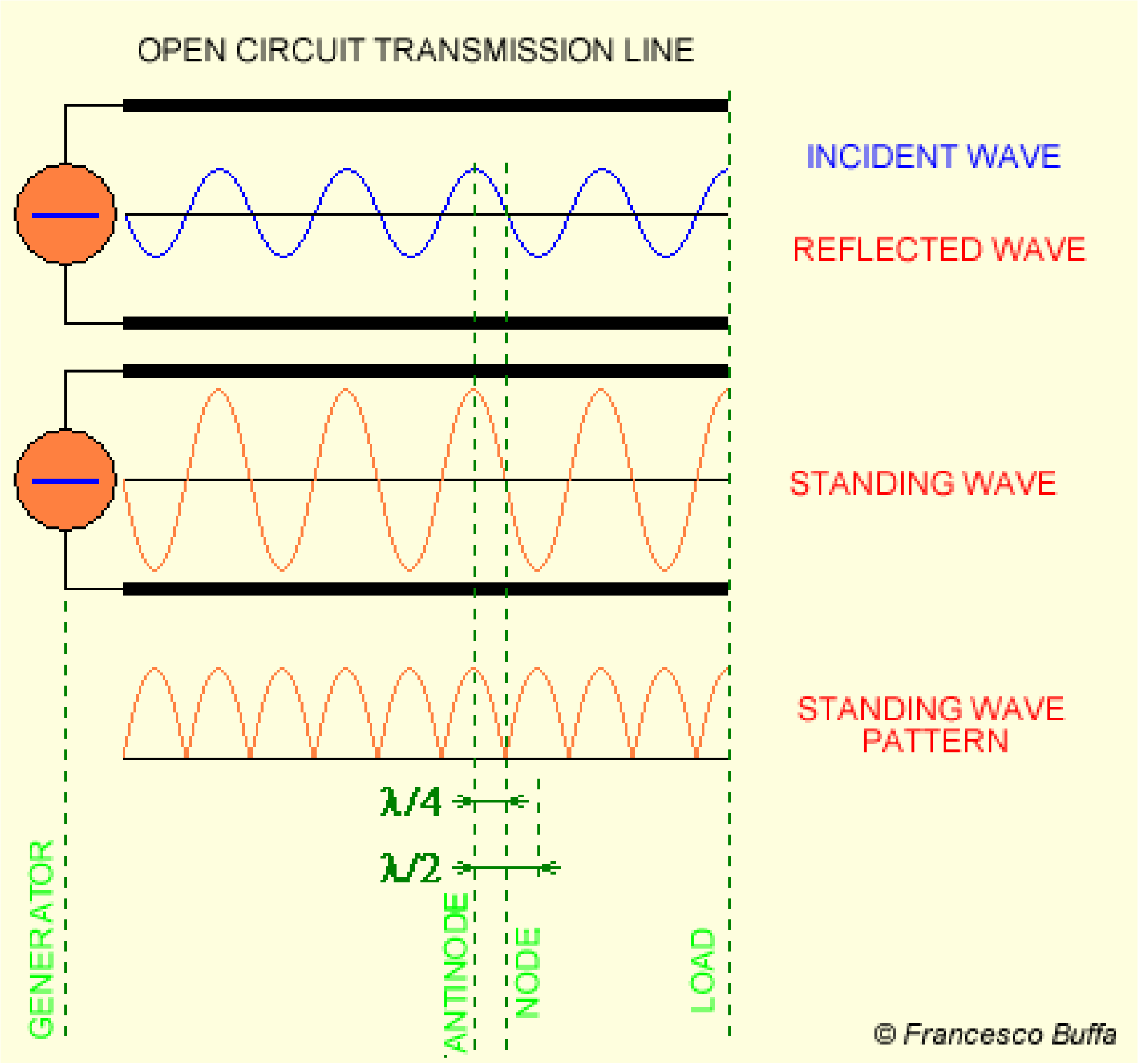



$$\text{VSWR} = \frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}$$

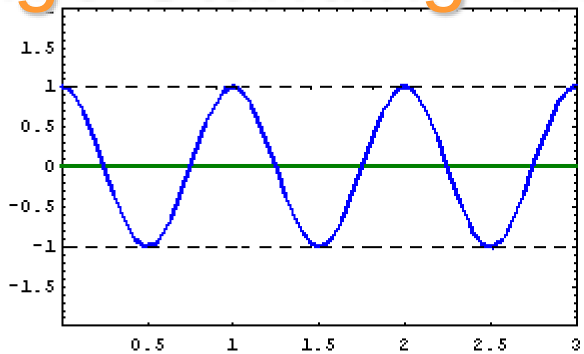
# Voltage Standing Wave Ratio



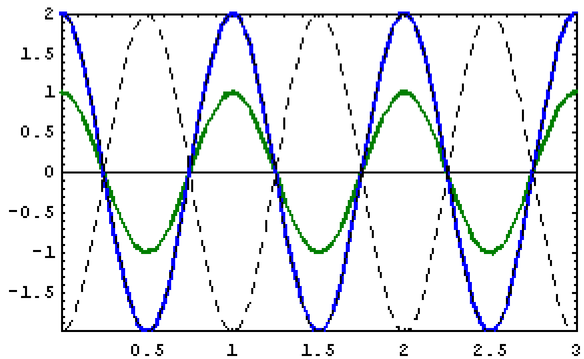
# Voltage Standing Wave Ratio

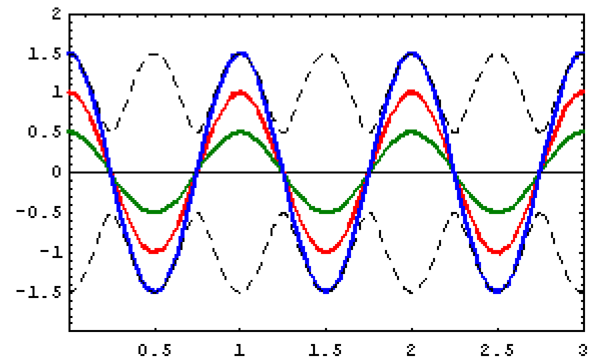


# Voltage Standing Wave Ratio



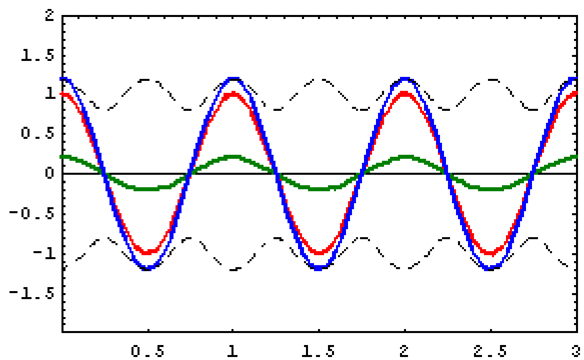

VSWR=(1+0)/|1-0|=1




VSWR=(1+1)/|1-1|=∞

VSWR=(1+0.5)/|1-0.5|=3

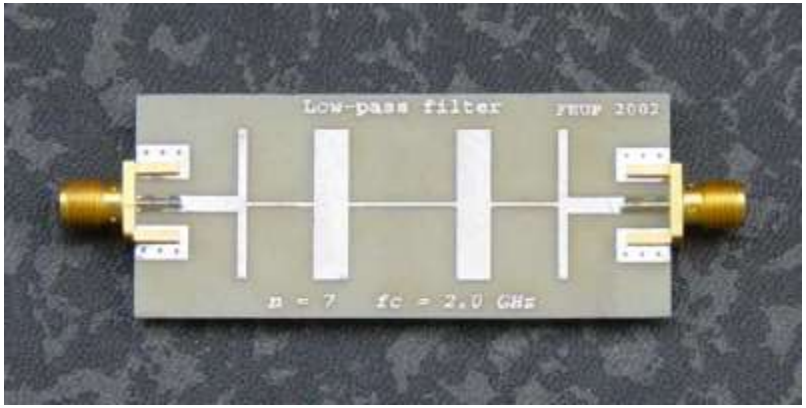



VSWR=(1+0.2)/|1-0.2|=1.5

# Using Transmission Lines to Synthesize Loads

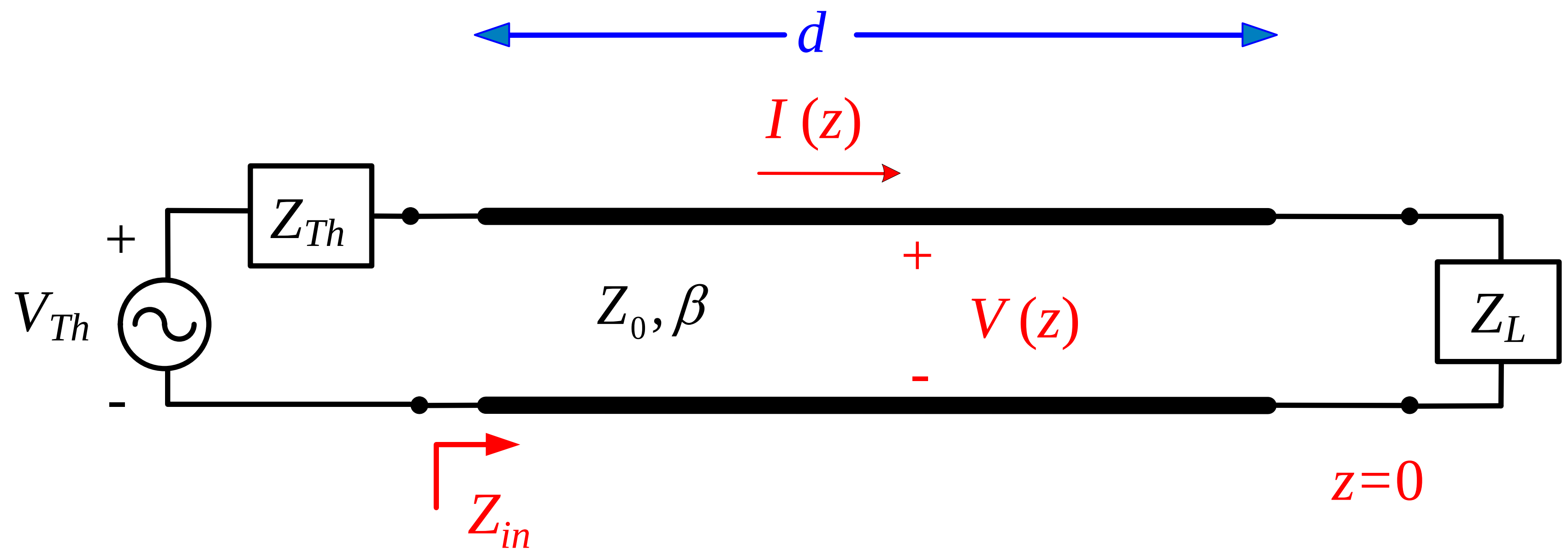
We can obtain any reactance that we want from a short or open transmission line.

This is very useful in microwave engineering.




A microwave filter constructed from microstrip line.

# Voltage on a Transmission Line

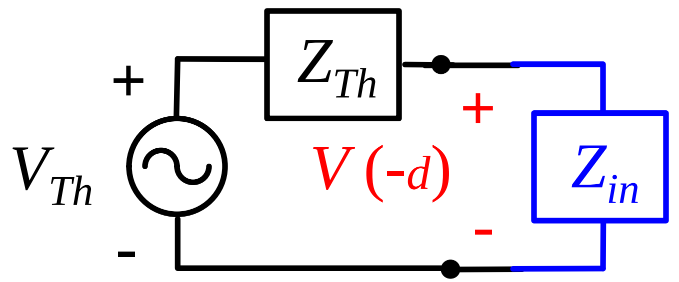



Find the voltage at any point on the line.

At the input:

$$V(-d) = V_{Th}\left(\frac{Z_{in}}{Z_{in}+Z_{Th}}\right)$$

$$Z_{in} = Z(-d) = Z_0\left(\frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}\right)$$



# Voltage on a Transmission Line (cont.)

$$V(z) = V_0^+ e^{-j\beta z}\left(1+\Gamma_L e^{+2j\beta z}\right) \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

At $z = -d$:

$$V(-d) = V_0^+ e^{j\beta d}\left(1+\Gamma_L e^{-j2\beta d}\right) = V_{Th}\left(\frac{Z_{in}}{Z_{in}+Z_{Th}}\right)$$

$$\Rightarrow V_0^+ = V_{Th}\left(\frac{Z_{in}}{Z_{in}+Z_{Th}}\right) e^{-j\beta d}\left(\frac{1}{1+\Gamma_L e^{-j2\beta d}}\right)$$

Hence

$$V(z) = V_{Th}\left(\frac{Z_{in}}{Z_{in}+Z_{Th}}\right) e^{-j\beta(d+z)}\left(\frac{1+\Gamma_L e^{+j2\beta z}}{1+\Gamma_L e^{-j2\beta d}}\right)$$

# Voltage on a Transmission Line (cont.)

Let's derive an alternative form of the result.

Start with: $$Z_{in} = Z(-d) = Z_0 \left( \frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}} \right)$$

$$\Rightarrow \frac{Z_{in}}{Z_{in}+Z_{Th}} = \frac{Z_0 \left( \dfrac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}} \right)}{Z_0 \left( \dfrac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}} \right) + Z_{Th}} = \frac{Z_0 \left(1+\Gamma_L e^{-j2\beta d}\right)}{Z_0 \left(1+\Gamma_L e^{-j2\beta d}\right) + Z_{Th} \left(1-\Gamma_L e^{-j2\beta d}\right)}$$

$$= \frac{Z_0 \left(1+\Gamma_L e^{-j2\beta d}\right)}{\left(Z_{Th}+Z_0\right) + \Gamma_L e^{-j2\beta d} \left(Z_0 - Z_{Th}\right)}$$

$$= \left( \frac{Z_0}{Z_{Th}+Z_0} \right) \frac{\left(1+\Gamma_L e^{-j2\beta d}\right)}{1+\Gamma_L e^{-j2\beta d} \left( \dfrac{Z_0 - Z_{Th}}{Z_{Th}+Z_0} \right)}$$

$$= \left( \frac{Z_0}{Z_{Th}+Z_0} \right) \frac{\left(1+\Gamma_L e^{-j2\beta d}\right)}{1-\Gamma_L e^{-j2\beta d} \left( \dfrac{Z_{Th} - Z_0}{Z_{Th}+Z_0} \right)}$$

# Voltage on a Transmission Line (cont.)

Hence, we have

$$\frac{Z_{in}}{Z_{in}+Z_{Th}}=\left(\frac{Z_0}{Z_0+Z_{Th}}\right)\left(\frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_s\Gamma_L e^{-j2\beta d}}\right)$$

where $\Gamma_s \equiv \frac{Z_{Th}-Z_0}{Z_{Th}+Z_0}$ (source reflection coefficient)

Substitute

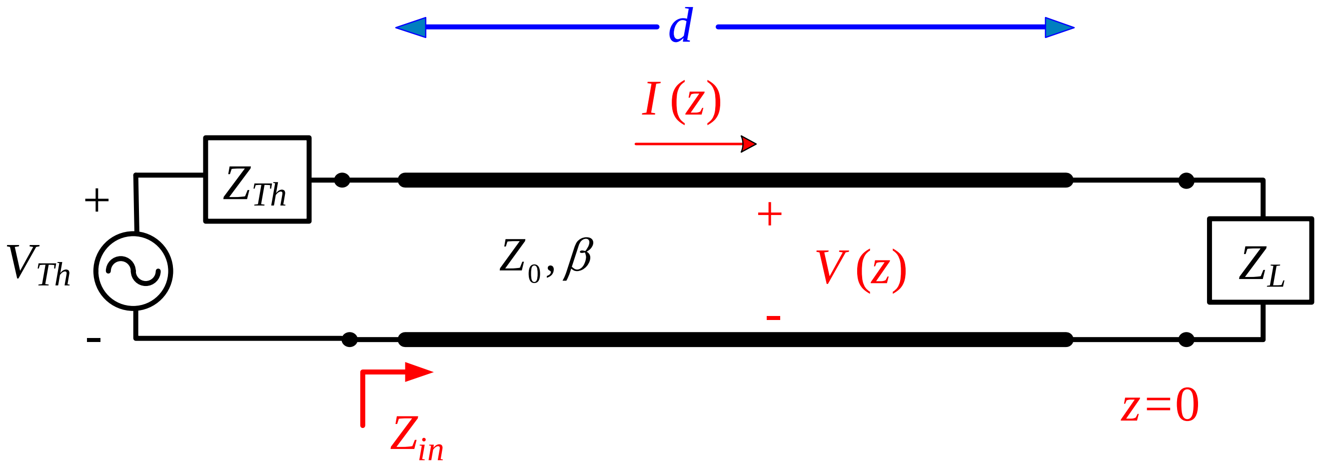
Recall:

$$V(z)=V_{Th}\left(\frac{Z_{in}}{Z_{in}+Z_{Th}}\right)e^{-j\beta(d+z)}\left(\frac{1+\Gamma_L e^{+j2\beta z}}{1+\Gamma_L e^{-j2\beta d}}\right)$$

Therefore, we have the following alternative form for the result:

$$V(z)=V_{Th}\left(\frac{Z_0}{Z_0+Z_{Th}}\right)e^{-j\beta(d+z)}\left(\frac{1+\Gamma_L e^{+j2\beta z}}{1-\Gamma_s\Gamma_L e^{-j2\beta d}}\right)$$

# Voltage on a Transmission Line (cont.)

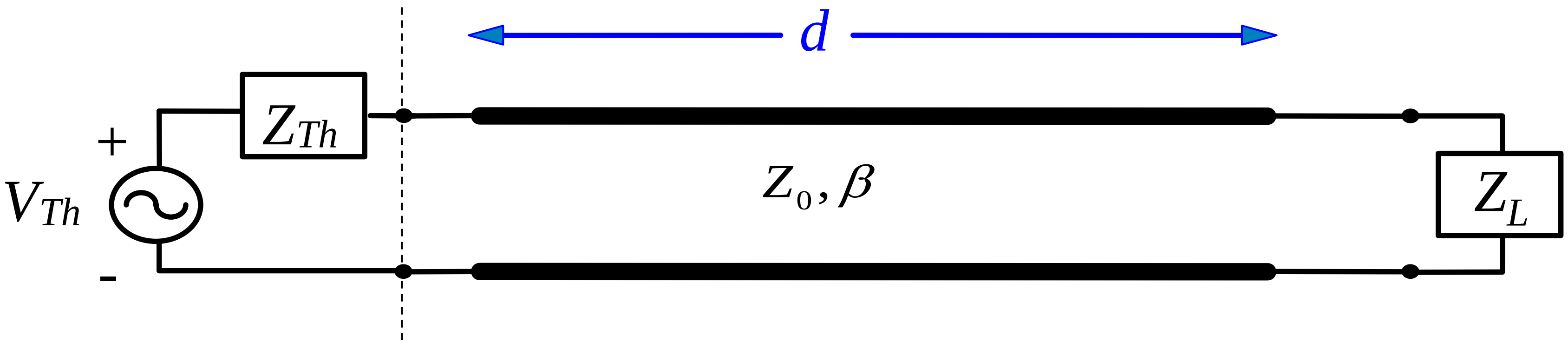



$$V(z) = V_{Th}\left(\frac{Z_0}{Z_0 + Z_{Th}}\right)e^{-j\beta(z-(-d))}\left(\frac{1+\Gamma_L e^{+j2\beta z}}{1-\Gamma_s \Gamma_L e^{-j2\beta d}}\right)$$

This term accounts for the multiple (infinite) bounces.

Voltage wave that would exist if there were no reflections from the load (a semi-infinite transmission line or a matched load).

# Voltage on a Transmission Line (cont.)




Wave-bounce method (illustrated for $z = -d$):

$$V(-d) = V_{Th}\left(\frac{Z_0}{Z_0 + Z_{Th}}\right)\left[\begin{array}{l} 1 + \Gamma_L e^{-j2\beta d} + \left(\Gamma_L e^{-j2\beta d}\right)\Gamma_s \\ \quad + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_s\right]\left(\Gamma_L e^{-j2\beta d}\right) + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_s\left(\Gamma_L e^{-j2\beta d}\right)\right]\Gamma_s \\ \qquad + \ldots \end{array}\right]$$

# Voltage on a Transmission Line (cont.)

$$V(-d) = V_{Th}\left(\frac{Z_0}{Z_0 + Z_{Th}}\right)\begin{bmatrix} 1 + \Gamma_L e^{-j2\beta d} + \left(\Gamma_L e^{-j2\beta d}\right)\Gamma_s \\ + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_s\right]\left(\Gamma_L e^{-j2\beta d}\right) + \left[\left(\Gamma_L e^{-j2\beta d}\right)\Gamma_s\left(\Gamma_L e^{-j2\beta d}\right)\right]\Gamma_s \\ + \ldots \end{bmatrix}$$

Group together alternating terms:

$$V(-d) = V_{Th}\left(\frac{Z_0}{Z_0 + Z_{Th}}\right)\begin{bmatrix} 1 + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right) + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right)^2 + \ldots \\ + \Gamma_L e^{-j2\beta d}\left[1 + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right) + \left(\Gamma_L \Gamma_S e^{-j2\beta d}\right)^2 + \ldots\right] \\ + \ldots \end{bmatrix}$$

Geometric series:

$$\sum_{n=0}^{\infty} z^n = 1 + z + z^2 + \ldots = \frac{1}{1-z}, \quad |z| < 1 \qquad\qquad z \equiv \Gamma_L \Gamma_s e^{-j2\beta d}$$

# Voltage on a Transmission Line (cont.)

Hence

$$V(-d) = V_{Th}\left(\frac{Z_0}{Z_0 + Z_{Th}}\right)\left[\frac{1}{1-\Gamma_L\Gamma_s e^{-j2\beta d}} + \Gamma_L e^{-j2\beta d}\left(\frac{1}{1-\Gamma_L\Gamma_s e^{-j2\beta d}}\right)\right]$$

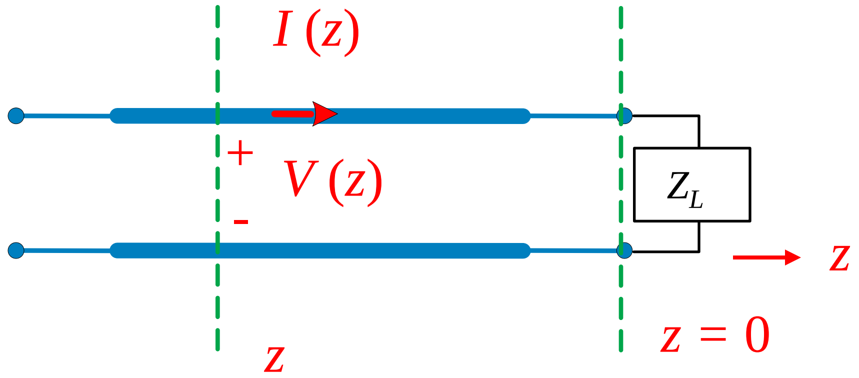
or

$$V(-d) = V_{Th}\left(\frac{Z_0}{Z_0 + Z_{Th}}\right)\left[\frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L\Gamma_s e^{-j2\beta d}}\right]$$

This agrees with the previous result (setting $z = -d$).

Note: The wave-bounce method is a *very tedious method* – not recommended.

# Time-Average Power Flow




At a distance $d$ from the load:

$$P(z) = \frac{1}{2}\operatorname{Re}\left\{V(z)\, I^*(z)\right\}$$

$$= \frac{1}{2}\operatorname{Re}\left[\frac{\left|V_0^+\right|^2}{Z_0^*} e^{-2\alpha z}\left(1+\Gamma_L e^{+2\gamma z}\right)\left(1-\Gamma_L^* e^{+2\gamma^* z}\right)\right]$$

$$V(z) = V_0^+ e^{-\gamma z}\left(1+\Gamma_L e^{+2\gamma z}\right)$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z}\left(1-\Gamma_L e^{+2\gamma z}\right)$$

$$\gamma = \alpha + j\beta$$

If $Z_0 \approx$ real (low-loss transmission line)

$$P(z) \approx \frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0} e^{-2\alpha z}\left(1-\left|\Gamma_L\right|^2 e^{+4\alpha z}\right)$$

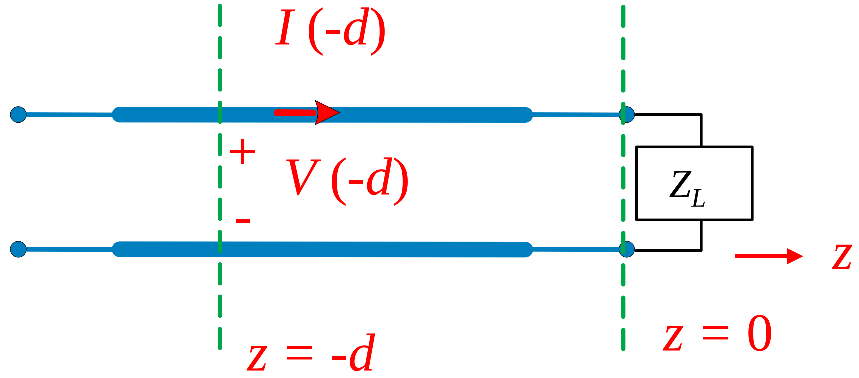
Note:

$$\Gamma_L e^{+2\gamma z} - \Gamma_L^* e^{+2\gamma^* z}$$

$$= \Gamma_L e^{+2\gamma z} - \left(\Gamma_L e^{+2\gamma z}\right)^*$$

$= \textit{pure imaginary}$

# Time-Average Power Flow (cont.)




## Low-loss line

$$P(z) \approx \frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0} e^{-2\alpha z}\left(1-\left|\Gamma_L\right|^2 e^{+4\alpha z}\right)$$

$$= \underbrace{\frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0} e^{-2\alpha z}}_{\text{Power in forward-traveling wave}} - \underbrace{\frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0}\left|\Gamma_L\right|^2 e^{+2\alpha z}}_{\text{Power in backward-traveling wave}}$$

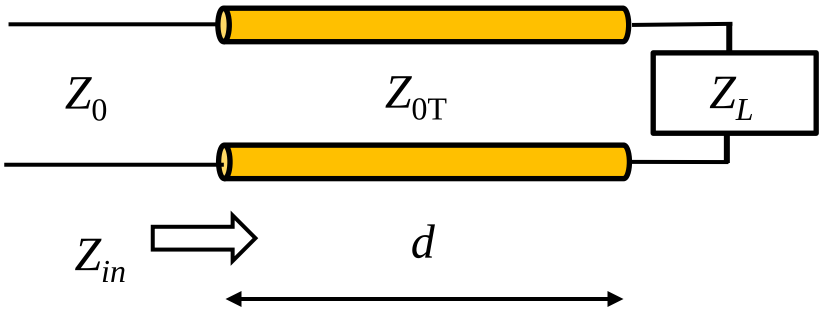
## Lossless line ($\alpha = 0$)

$$P(z) = \frac{1}{2}\frac{\left|V_0^+\right|^2}{Z_0}\left(1-\left|\Gamma_L\right|^2\right)$$

> Note:
> In the general lossy case, we cannot say that the total power is the difference of the powers in the two waves.

# Quarter-Wave Transformer

$$Z_{in} = Z_{0T}\left(\frac{Z_L + jZ_{0T}\tan\beta d}{Z_{0T} + jZ_L\tan\beta d}\right)$$




$$\beta d = \beta\frac{\lambda_g}{4} = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$$

$$\Rightarrow Z_{in} = Z_{0T}\left(\frac{jZ_{0T}}{jZ_L}\right)$$

so

$$Z_{in} = \frac{Z_{0T}^2}{Z_L}$$

Matching condition

$$\Gamma_{in} = 0 \quad \Rightarrow \quad Z_{in} = Z_0$$

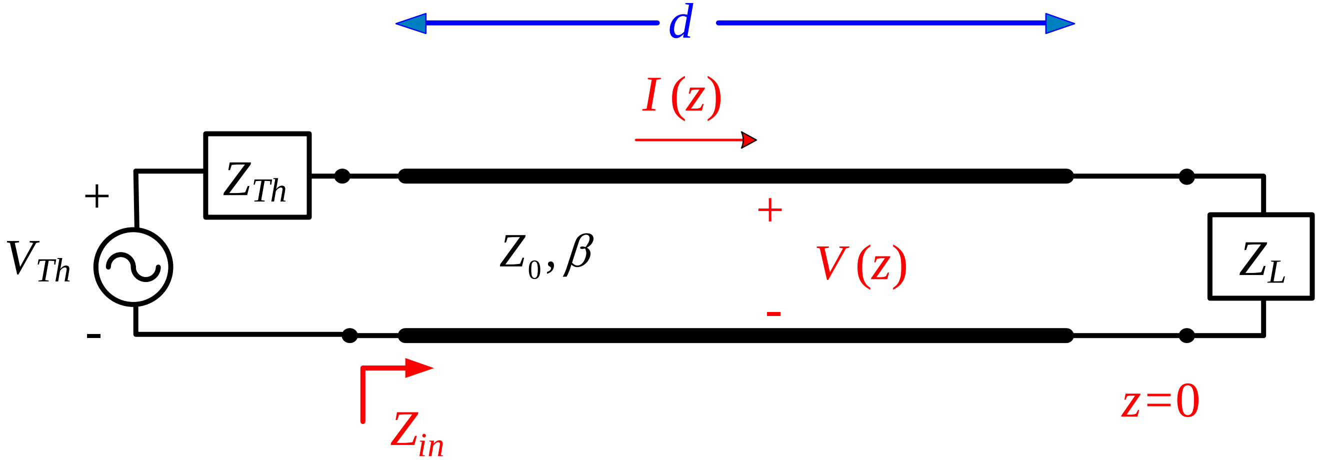
$$\Rightarrow \quad Z_0 = \frac{Z_{0T}^2}{Z_L}$$

(This requires $Z_L$ to be real.)

Hence

$$Z_{0T} = \sqrt{Z_0 Z_L}$$

# Conjugate matching




Home work

Prove that the condition for maximum power delivered to the load is

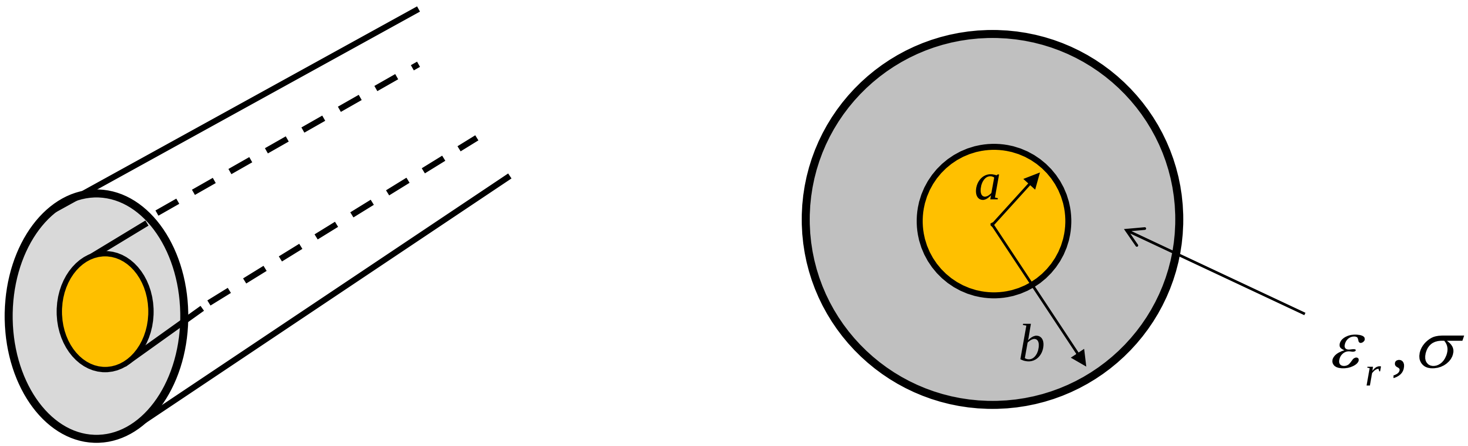
$$Z_{in} = Z_{th}^{*}$$

# Field Analysis of Transmission Lines

In this section we will derive the transmission line parameters (R, L, G, C) in terms of the electric and magnetic fields of the transmission line

# Coaxial Cable

Here we present a "case study" of one particular transmission line, the coaxial cable.

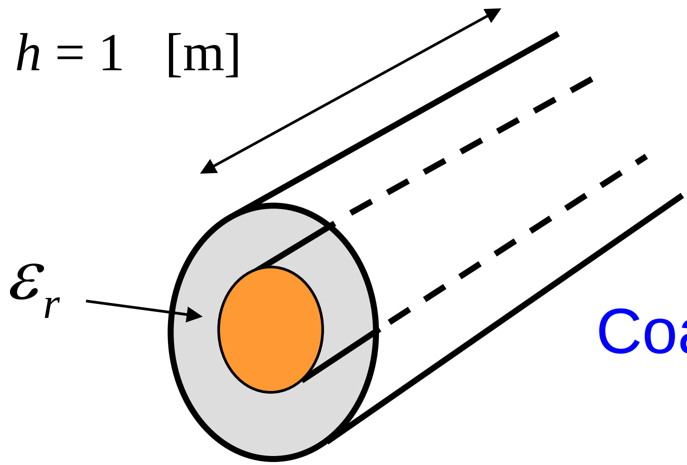



Find $C, L, G, R$

For a $\text{TEM}_z$ mode, the shape of the fields is independent of frequency, and hence we can perform the calculation using electrostatics and magnetostatics.

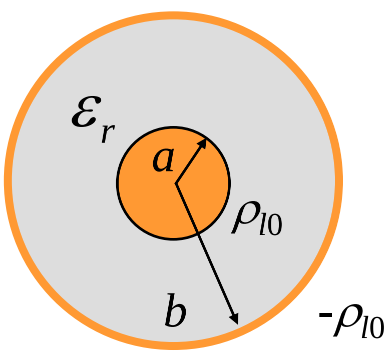
We will assume no variation in the $z$ direction, and take a length of one meter in the $z$ direction in order to calculate the per-unit-length parameters.

# Coaxial Cable (cont.)




Coaxial cable

Find $C$ (capacitance / length)



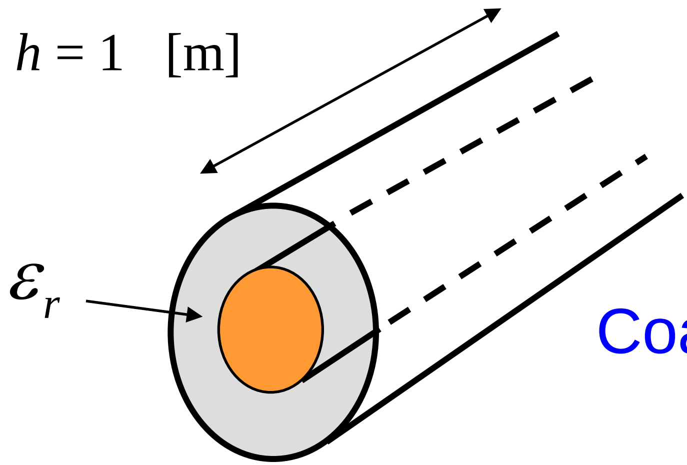

From Gauss's law:

$$\underline{E} = \underline{\hat{\rho}}\left(\frac{\rho_{\ell 0}}{2\pi\varepsilon\rho}\right) = \underline{\hat{\rho}}\left(\frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r\rho}\right)$$

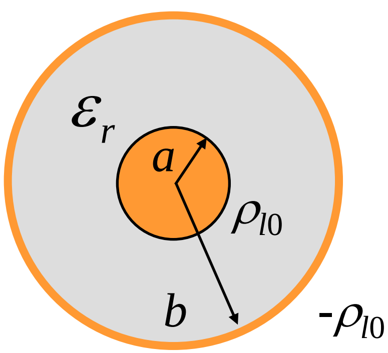
$$V = V_{AB} = \int_{\underline{A}}^{\underline{B}} \underline{E} \cdot \underline{dr}$$

$$= \int_a^b E_\rho \, d\rho = \frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r} \ln\left(\frac{b}{a}\right)$$

# Coaxial Cable (cont.)




Coaxial cable




Hence

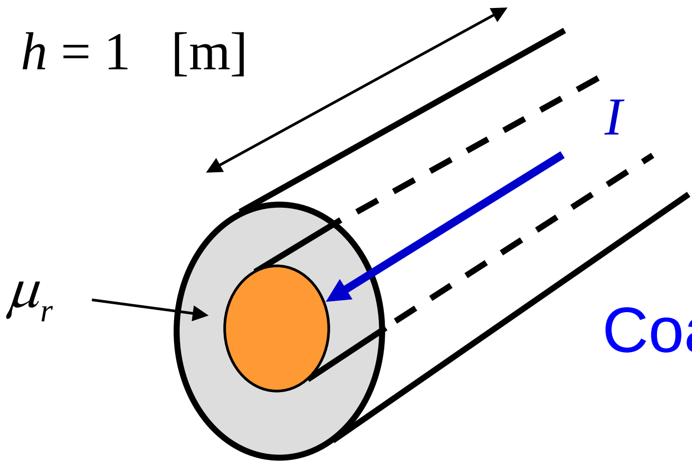
$$C = \frac{Q}{V} = \frac{\rho_{\ell 0}(1)}{\left(\dfrac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r}\right)\ln\left(\dfrac{b}{a}\right)}$$

We then have:

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\dfrac{b}{a}\right)} \quad [\text{F/m}]$$

# Coaxial Cable (cont.)

**Find $L$ (inductance / length)**




Coaxial cable

From Ampere's law:

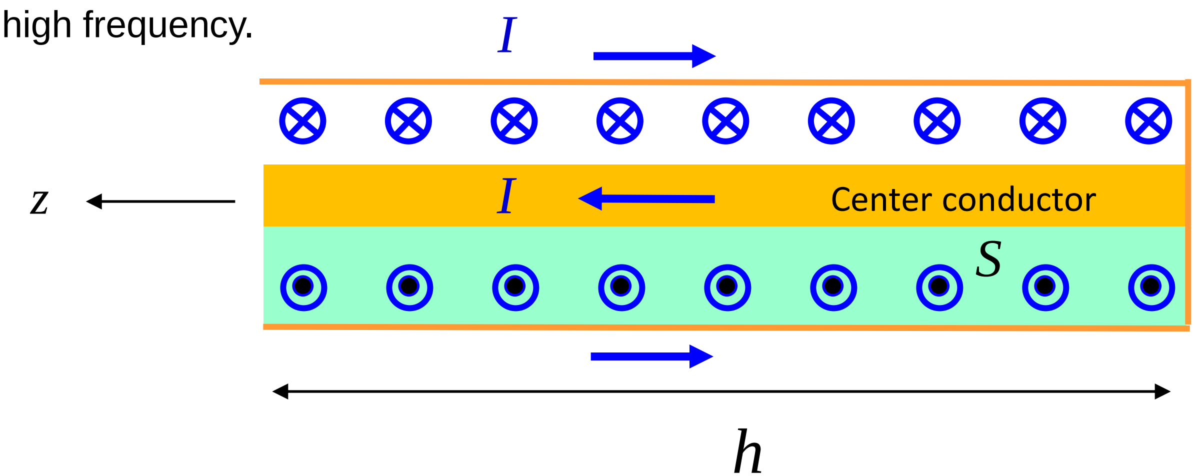
$$\underline{H} = \underline{\hat{\phi}} \left( \frac{I}{2\pi\rho} \right)$$

$$\underline{B} = \underline{\hat{\phi}} \left( \frac{I}{2\pi\rho} \right) \mu_0 \mu_r$$

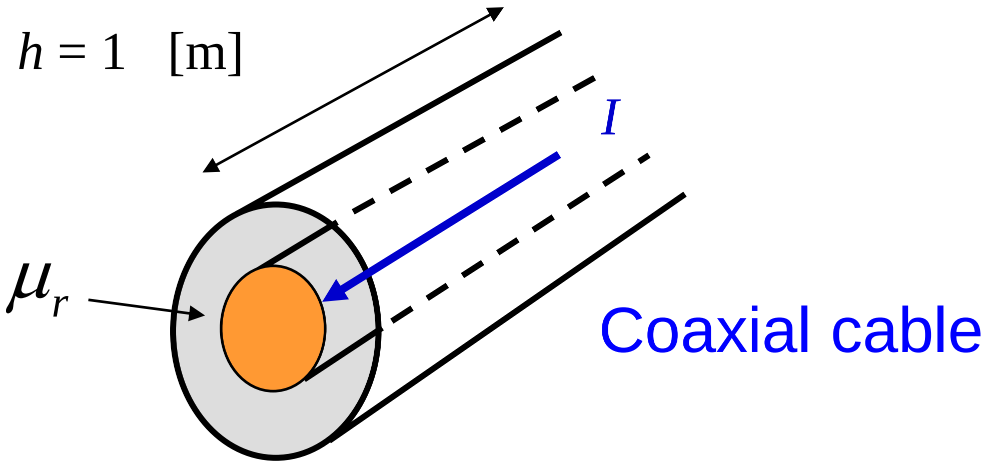
Note: We ignore "internal inductance" here, and only look at the magnetic field *between* the two conductors (accurate for high frequency.

Magnetic flux:

$$\psi = (1) \int_a^b B_\phi \, d\rho$$



# Coaxial Cable (cont.)




$$\psi = (1)\,\mu_0\mu_r \int_a^b H_\phi \, d\rho$$

$$= \mu_0\mu_r \int_a^b \frac{I}{2\pi\rho} \, d\rho$$

$$= \mu_0\mu_r \frac{I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$L = \frac{\psi}{I} = \mu_0\mu_r \frac{1}{2\pi} \ln\left(\frac{b}{a}\right)$$

Hence

$$L = \frac{\mu_0\mu_r}{2\pi} \ln\left(\frac{b}{a}\right) \quad [\mathrm{H/m}]$$

# Coaxial Cable (cont.)

Observation:

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$

$$LC = \mu\varepsilon = \mu_0\varepsilon_0\left(\mu_r\varepsilon_r\right)$$

This result actually holds for any transmission line (proof omitted).

# Coaxial Cable (cont.)

For a lossless cable: $$Z_0 = \sqrt{\frac{L}{C}}$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{F/m}]$$

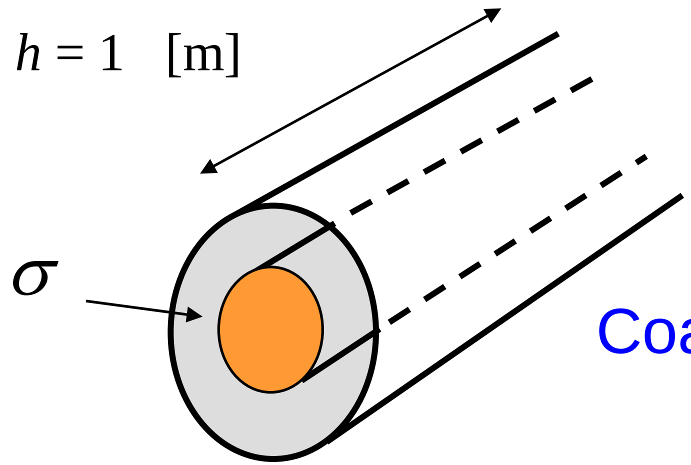
$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\mathrm{H/m}]$$

$$Z_0 = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}\frac{1}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\Omega]$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7303\ [\Omega]$$

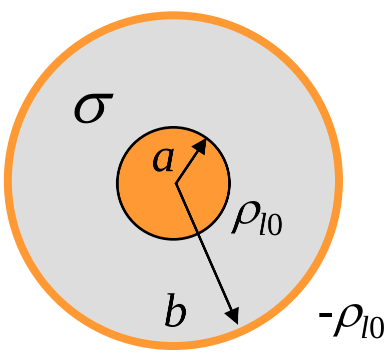
# Coaxial Cable (cont.)

$h = 1 \quad [\mathrm{m}]$




Coaxial cable

Find $G$ (conductance / length)

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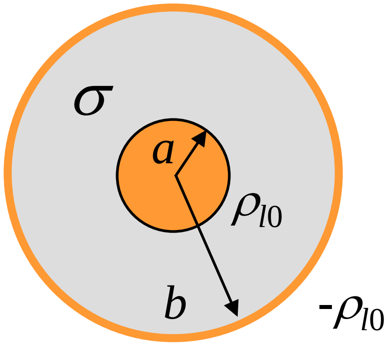
$$V = V_{AB} = \int_{\underline{A}}^{\underline{B}} \underline{E}\cdot\underline{dr}$$

$$= \int_a^b E_\rho\, d\rho = \frac{\rho_{\ell 0}}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{b}{a}\right)$$

# Coaxial Cable (cont.)




$$\underline{J} = \sigma \underline{E}$$

$$I_{leak} = J_\rho \big|_{\rho=a} \left[(1)\, 2\pi a\right]$$
$$= 2\pi a \sigma\, E_\rho \big|_{\rho=a}$$
$$= 2\pi a \sigma \left( \frac{\rho_{\ell 0}}{2\pi \varepsilon_0 \varepsilon_r a} \right)$$

We then have $G = \dfrac{I_{leak}}{V}$

$$G = \frac{2\pi a \sigma \left( \dfrac{\rho_{\ell 0}}{2\pi \varepsilon_0 \varepsilon_r a} \right)}{\dfrac{\rho_{\ell 0}}{2\pi \varepsilon_0 \varepsilon_r} \ln\left( \dfrac{b}{a} \right)}$$

or

$$G = \frac{2\pi \sigma}{\ln\left( \dfrac{b}{a} \right)} \quad [\mathrm{S/m}]$$

# Coaxial Cable (cont.)

Observation:

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{F/m}] \qquad \varepsilon = \varepsilon_0 \varepsilon_r$$

$$G = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{S/m}]$$

$$G = C\left(\frac{\sigma}{\varepsilon}\right)$$

This result actually holds for any transmission line (proof omitted).

# Coaxial Cable (cont.)

As just derived,

$$G = C\left(\frac{\sigma}{\varepsilon}\right)$$

Hence:

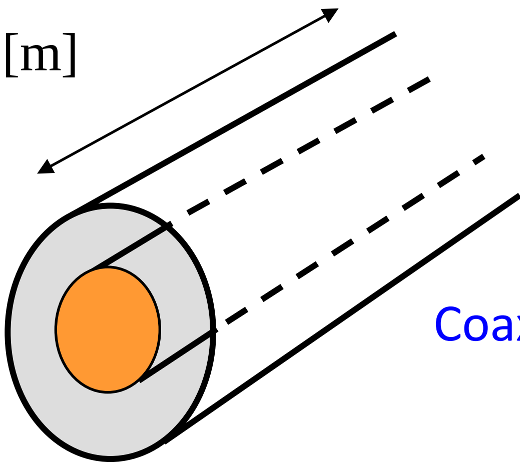
$$\frac{G}{\omega C} = \left(\frac{\sigma}{\omega\varepsilon}\right) = \tan\delta$$

This is the loss tangent that would arise from conductivity.

$$\frac{G}{\omega C} = \tan\delta$$

# Coaxial Cable (cont.)

$h = 1 \quad [\mathrm{m}]$




Find $R$ (resistance / length)

$$R = R_a + R_b$$

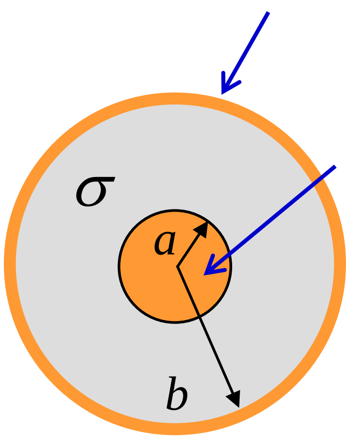
$$R_a = R_{sa}\left(\frac{1}{2\pi a}\right)$$

$$R_b = R_{sb}\left(\frac{1}{2\pi b}\right)$$

$R_s$ = surface resistance of metal

$\sigma_b, \mu_{rb}$

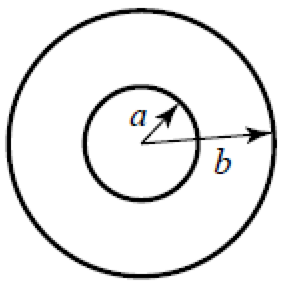
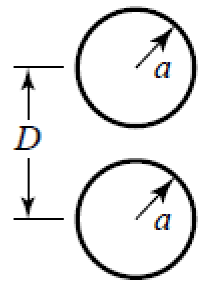
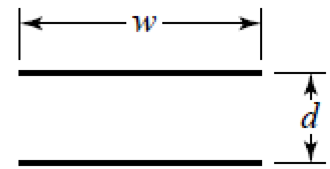
$\sigma_a, \mu_{ra}$




$$R_{sa} = \frac{1}{\sigma_a \delta_a} \qquad R_{sb} = \frac{1}{\sigma_b \delta_b}$$

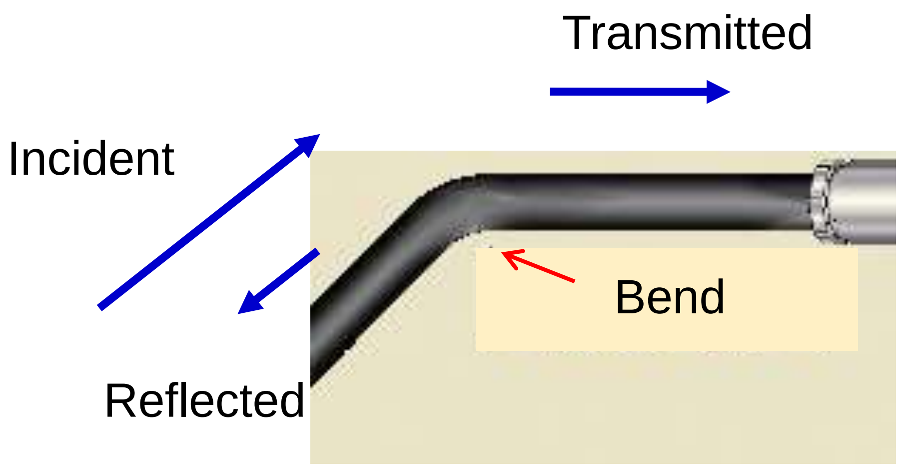
$$\delta_a = \sqrt{\frac{2}{\omega \mu_0 \mu_{ra} \sigma_a}} \qquad \delta_b = \sqrt{\frac{2}{\omega \mu_0 \mu_{rb} \sigma_b}}$$

**TABLE 2.1 Transmission Line Parameters for Some Common Lines**

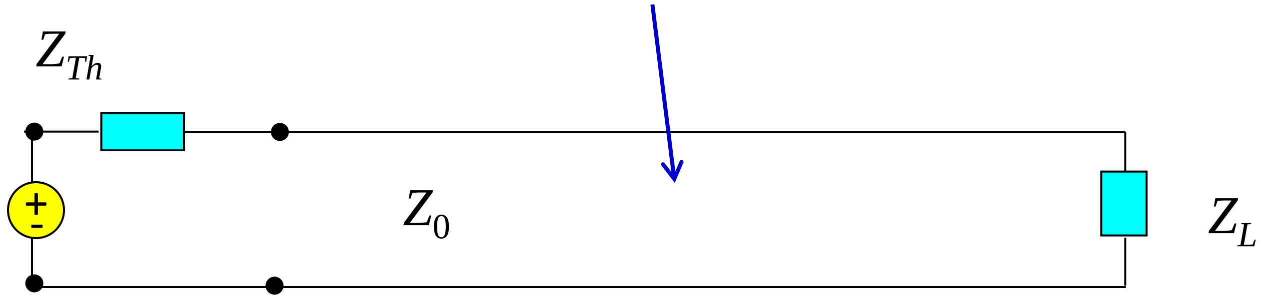
| | COAX | TWO-WIRE | PARALLEL PLATE |
|---|---|---|---|
| |  |  |  |
| $L$ | $\dfrac{\mu}{2\pi}\ln\dfrac{b}{a}$ | $\dfrac{\mu}{\pi}\cosh^{-1}\left(\dfrac{D}{2a}\right)$ | $\dfrac{\mu d}{w}$ |
| $C$ | $\dfrac{2\pi\epsilon'}{\ln b/a}$ | $\dfrac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$ | $\dfrac{\epsilon' w}{d}$ |
| $R$ | $\dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ | $\dfrac{R_s}{\pi a}$ | $\dfrac{2R_s}{w}$ |
| $G$ | $\dfrac{2\pi\omega\epsilon''}{\ln b/a}$ | $\dfrac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$ | $\dfrac{\omega\epsilon'' w}{d}$ |

# Limitations of Transmission-Line Theory

At high frequency, discontinuity effects can become important.



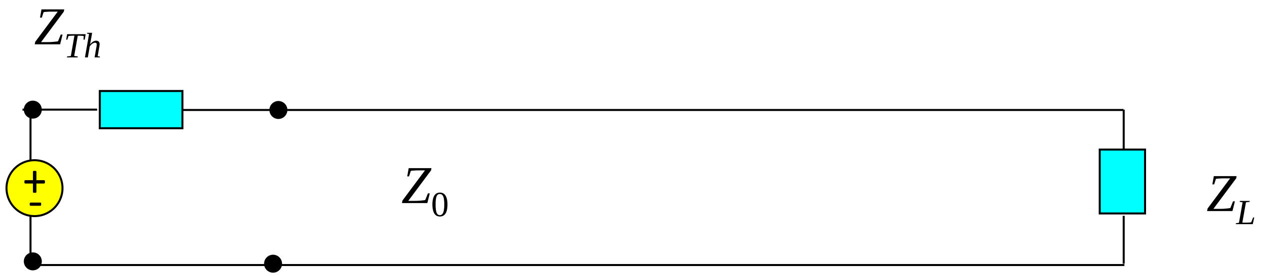

The simple TL model does not account for the bend.



# Limitations of Transmission-Line Theory (cont.)

At high frequency, radiation effects can become important.

We want energy to travel from the generator to the load, without radiating.




When will radiation occur?

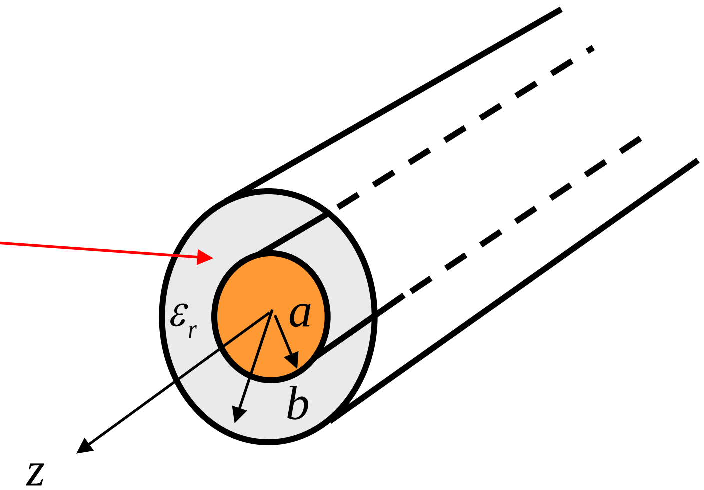
This is explored next.

# Limitations of Transmission-Line Theory (cont.)

## Coaxial Cable

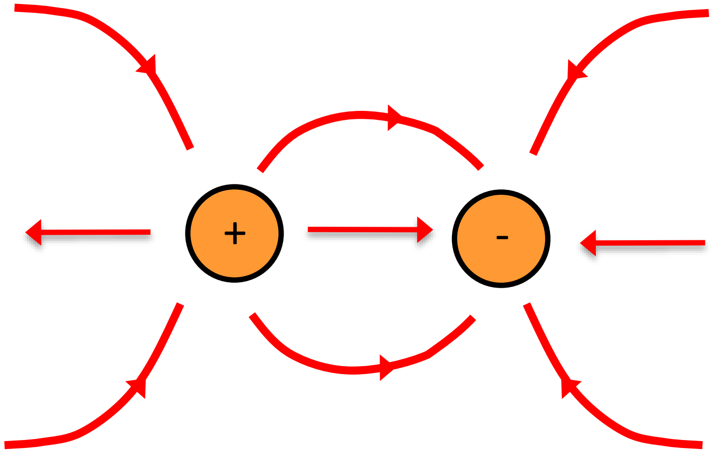
The coaxial cable is a perfectly shielded system – there is never any radiation at any frequency, as long as the metal thickness is large compared with a skin depth.

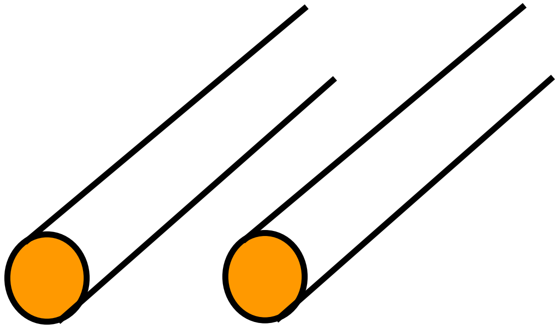
The fields are confined to the region between the two conductors.



# Limitations of Transmission-Line Theory (cont.)

The twin lead is an open type of transmission line – the fields extend out to infinity.



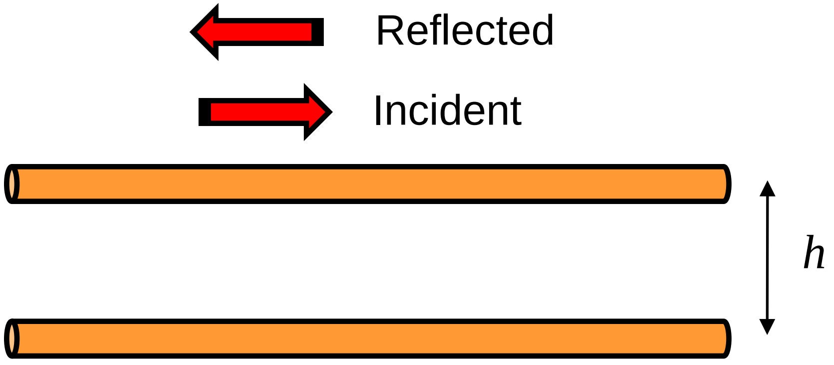
The extended fields may cause interference with nearby objects.
(This may be improved by using “twisted pair”.)

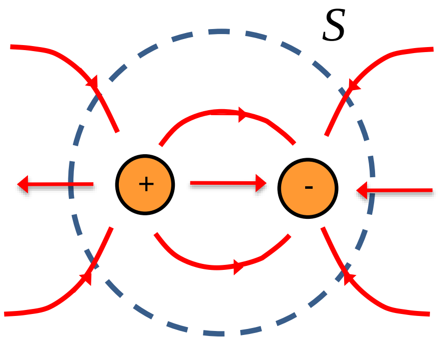
Having fields that extend to infinity is not the same thing as having radiation, however!

# Limitations of Transmission-Line Theory (cont.)

The infinite twin lead will not radiate by itself, regardless of how far apart the lines are (this is true for any transmission line).

$$P_t = \int_S \text{Re}\left(\frac{1}{2}\left(\underline{E} \times \underline{H}^*\right)\right) \cdot \underline{\hat{\rho}}\, dS = 0$$



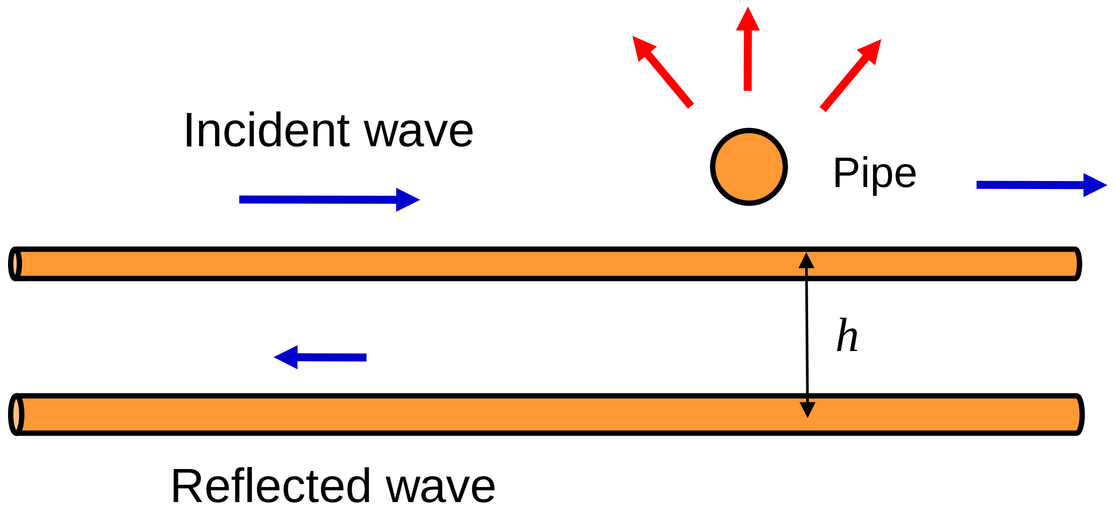

No attenuation on an infinite lossless line

The incident and reflected waves represent an exact solution to Maxwell's equations on the infinite line, at any frequency.

# Limitations of Transmission-Line Theory (cont.)

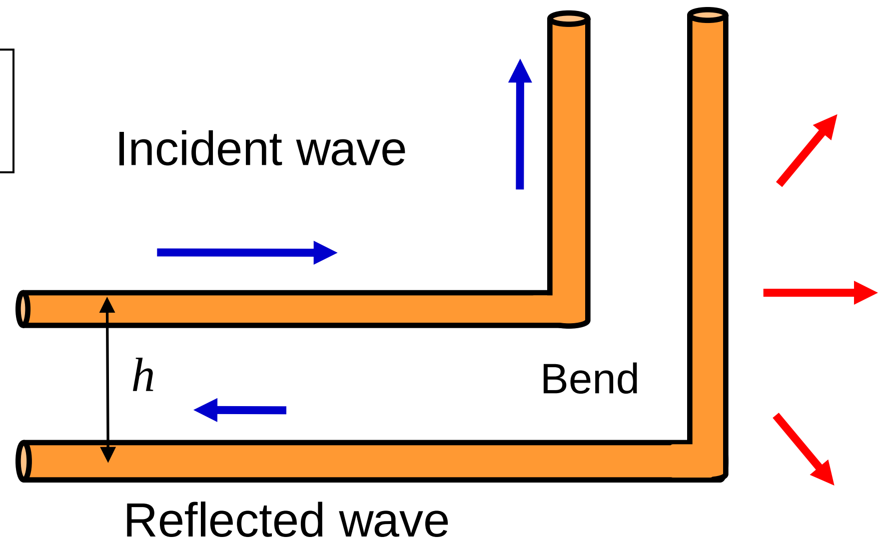
A discontinuity on the twin lead will cause radiation to occur.

Obstacle




Note: Radiation effects usually increase as the frequency increases.

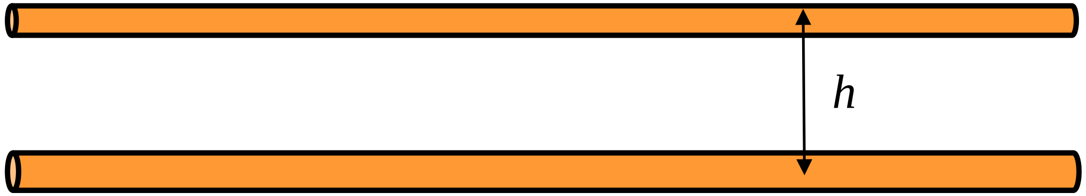
Bend



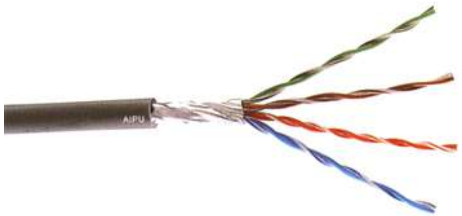
# Limitations of Transmission-Line Theory (cont.)

To reduce radiation effects of the twin lead at discontinuities:

1) Reduce the separation distance $h$ (keep $h << \lambda$).
2) Twist the lines (twisted pair).




CAT 5 cable
(twisted pair)



# Baluns

## Baluns are used to connect coaxial cables to twin leads.

**They suppress the common mode currents on the transmission lines.**

Balun: “Balanced to “unbalanced”

Coaxial cable: an “unbalanced” transmission line

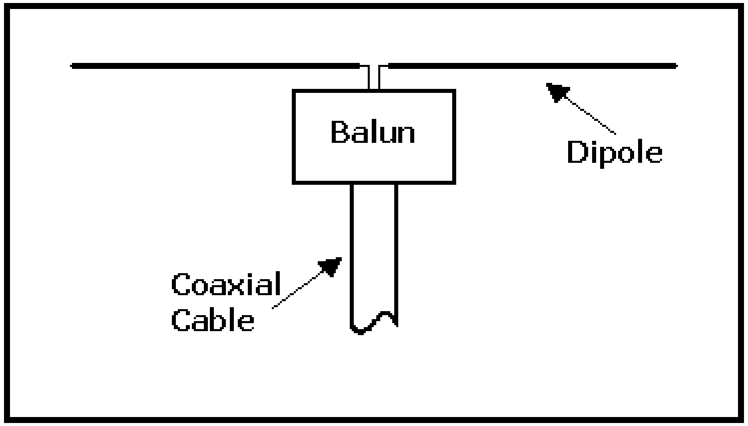
Twin lead: a “balanced” transmission line


4:1 impedance-transforming baluns, connecting 75Ω TV coax to 300Ω TV twin lead

https://en.wikipedia.org/wiki/Balun

# Baluns (cont.)

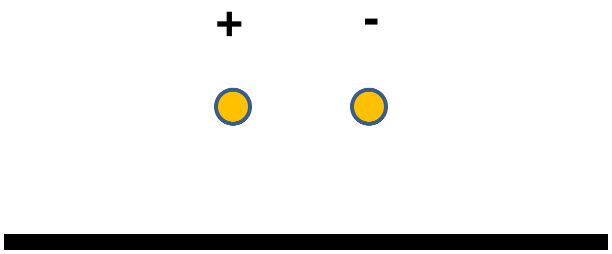
Baluns are also used to connect coax (unbalanced line) to dipole antennas (balanced).




**“Baluns are present in radars, transmitters, satellites, in every telephone network, and probably in most wireless network modem/routers used in homes.”**

https://en.wikipedia.org/wiki/Balun

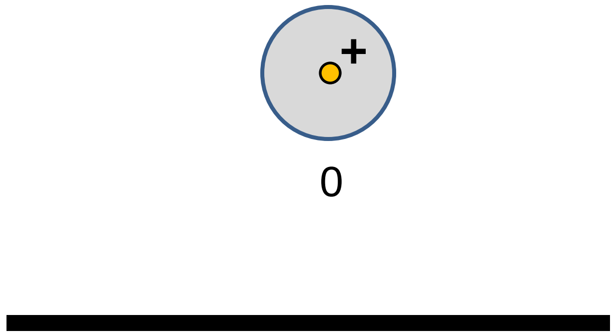
# Baluns (cont.)




Ground (zero volts)

**Twin Lead**

The voltages are *balanced* with respect to ground.




Ground (zero volts)

**Coax**

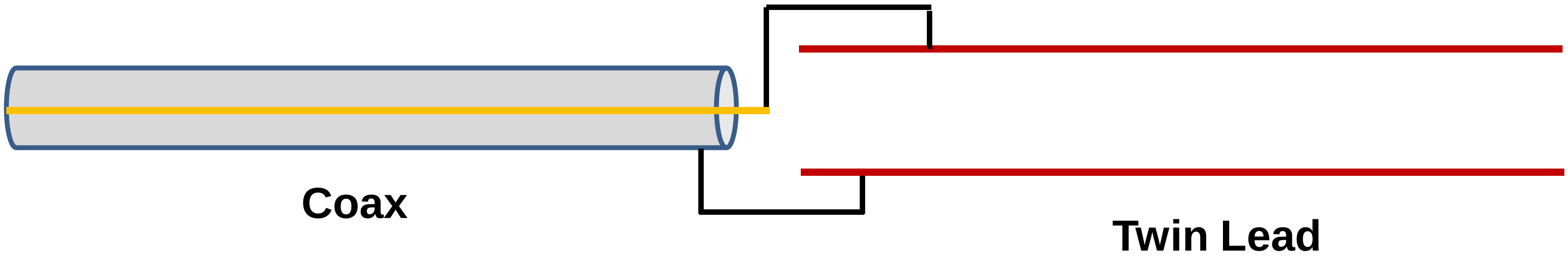
The voltages are *unbalanced* with respect to ground.

# Baluns (cont.)

Baluns are necessary because, in practice, the two transmission lines are always both running over a *ground plane*.

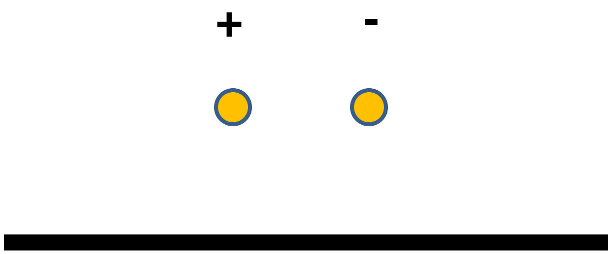
If there were no ground plane, and you only had the two lines connected to each other, then a balun would not be necessary.

(But you would still want to have a matching network between the two lines if they have different characteristic impedances.)



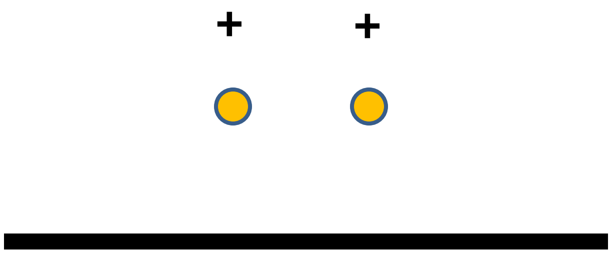
# Baluns (cont.)

When a ground plane is present, we really have three conductors, forming a *multiconductor* transmission line, and this system supports two different modes.




**Differential mode**

(desired)

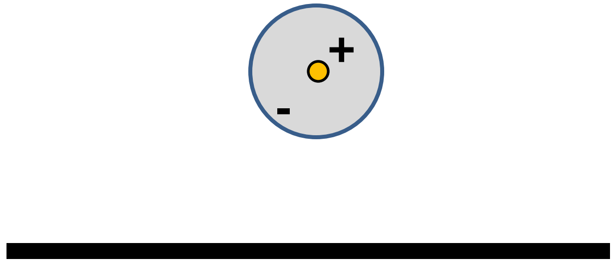



**Common mode**

(undesired)

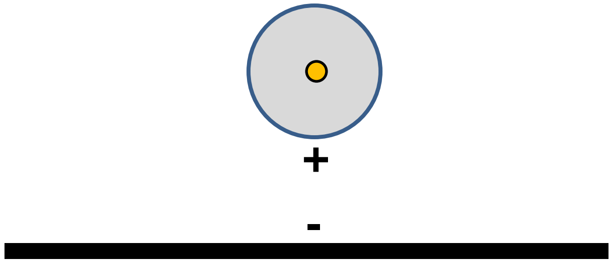
The differential and common modes on a twin lead over ground

# Baluns (cont.)




**Differential mode**

(desired)



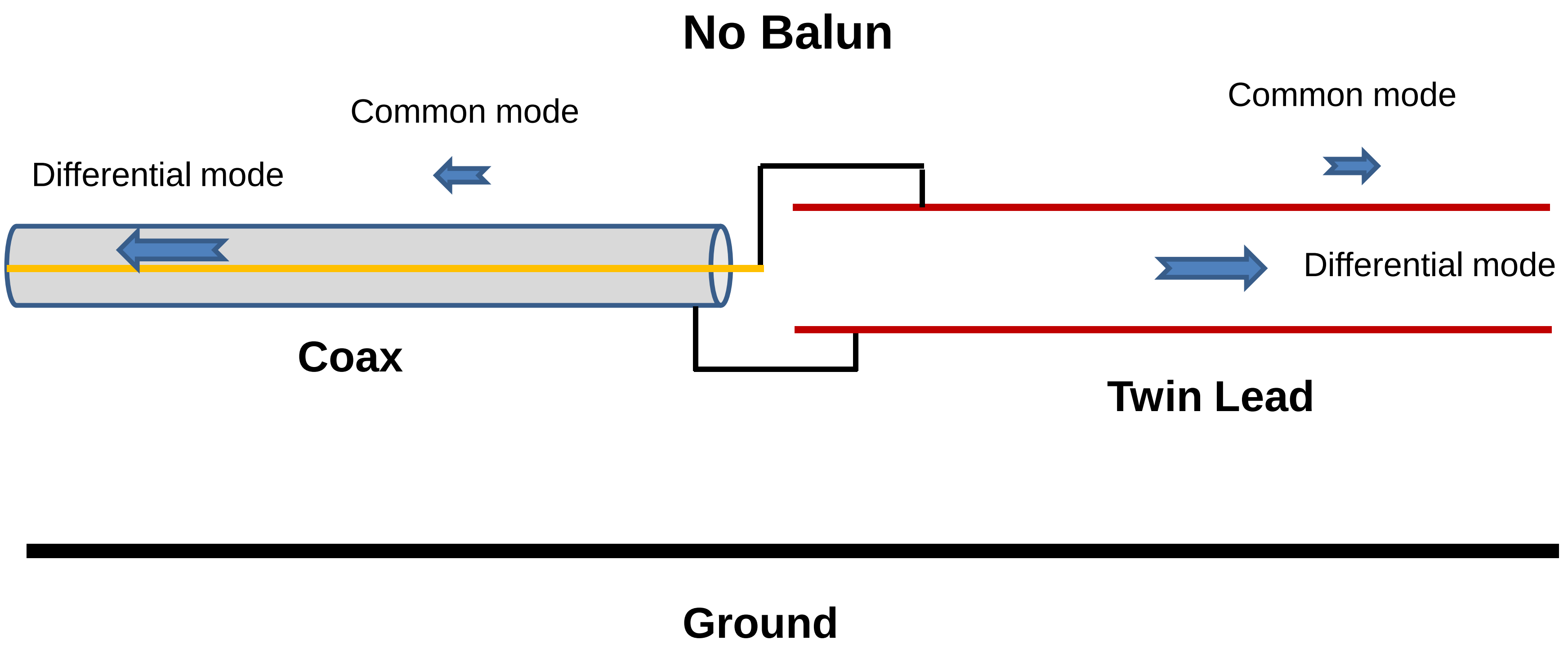

**Common mode**

(undesired)

The differential and common modes on a coax over ground

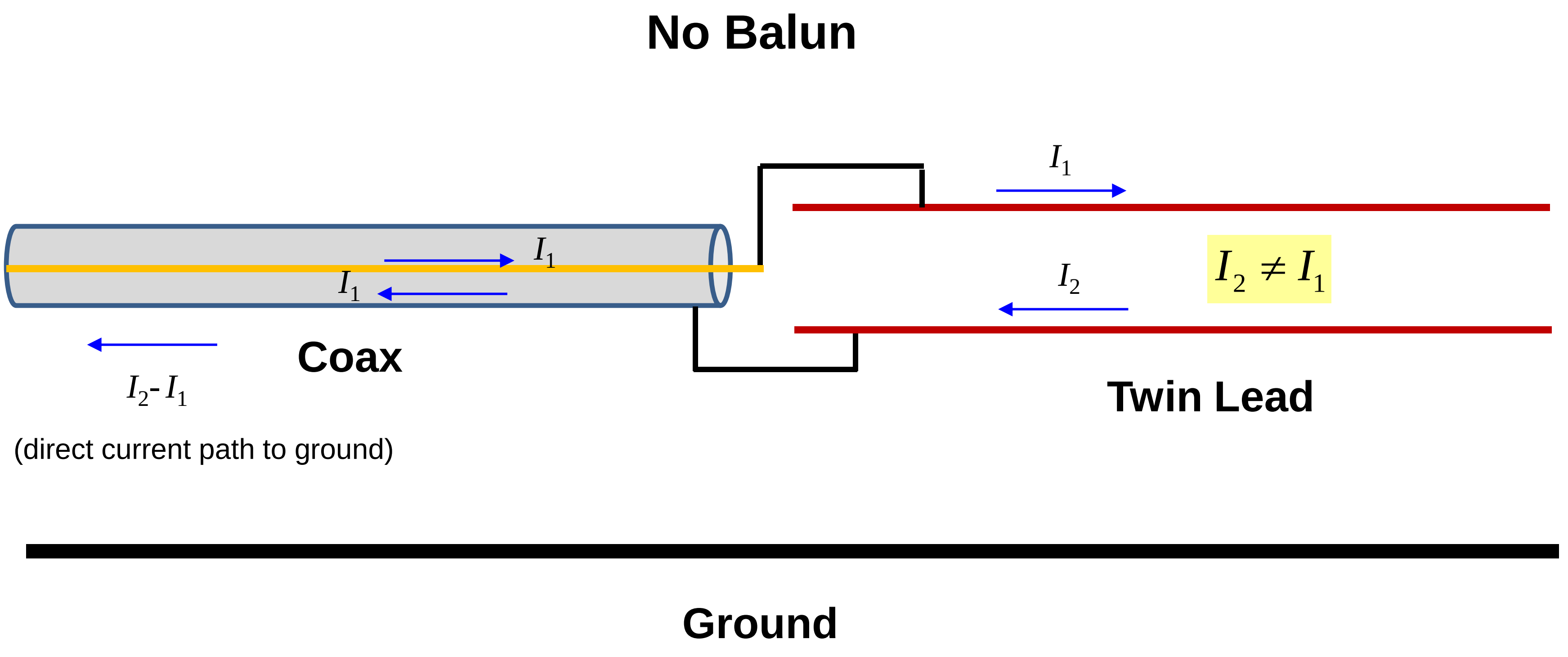
# Baluns (cont.)

A balun prevents common modes from being excited at the junction between a coax and a twin lead.



# Baluns (cont.)

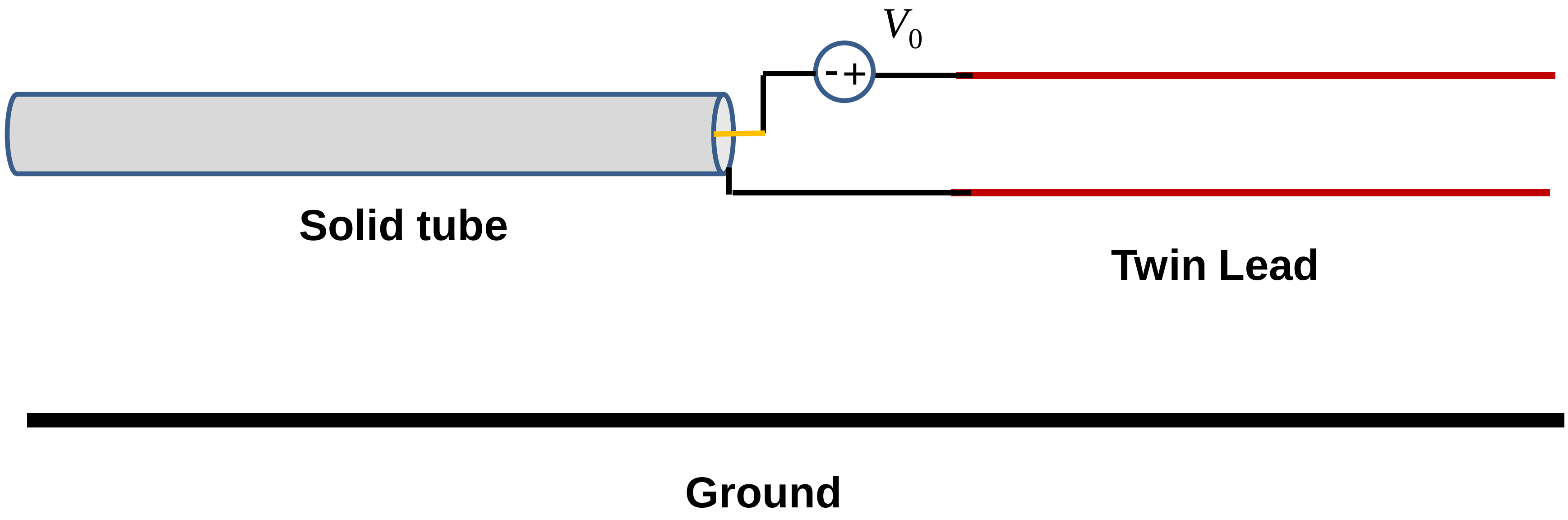
From another point of view, a balun prevents currents from flowing on the *outside of the coax*.



# Baluns (cont.)

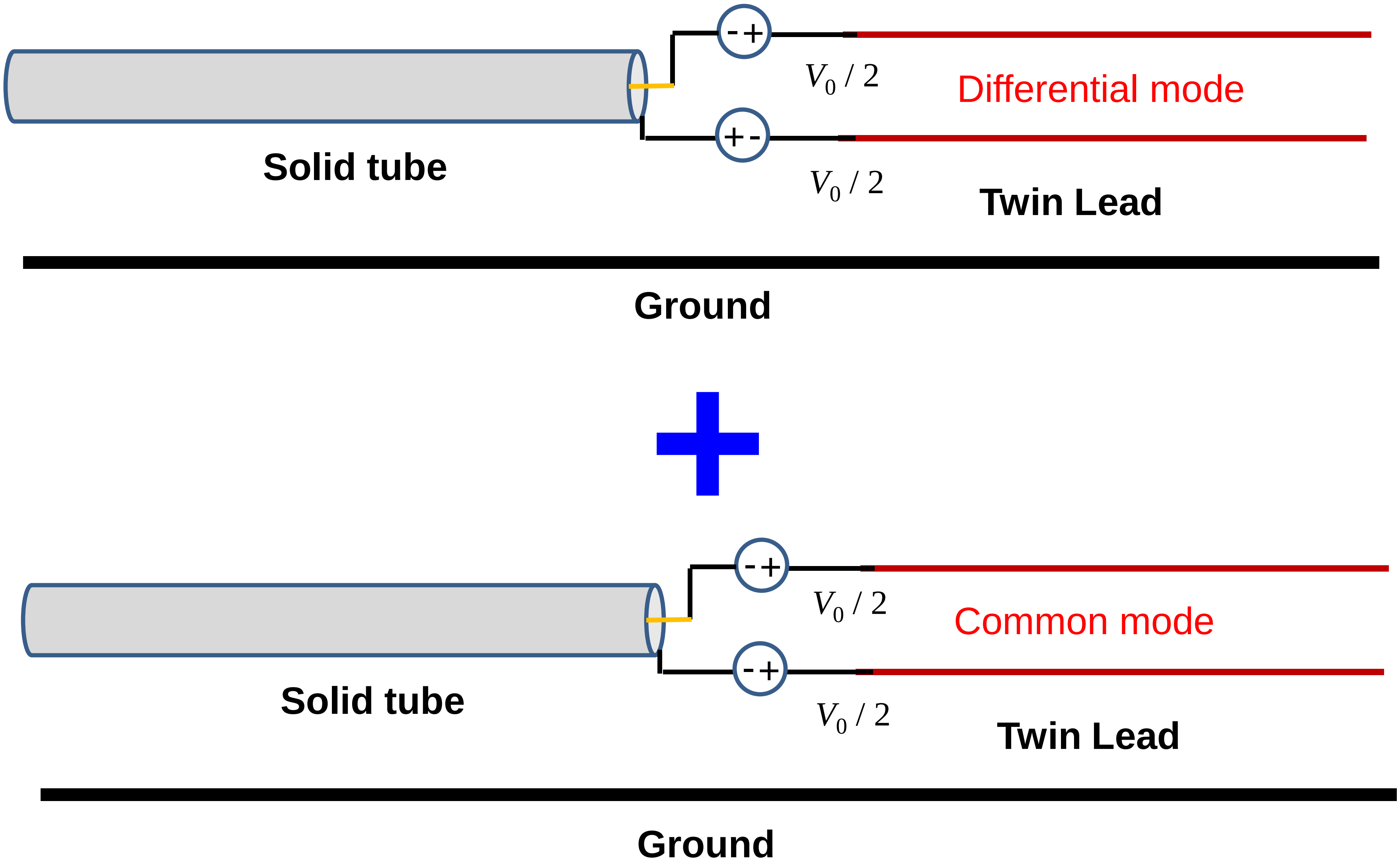
A simple model for the mode excitation at the junction

The coax is replaced by a solid tube with a voltage source at the end.



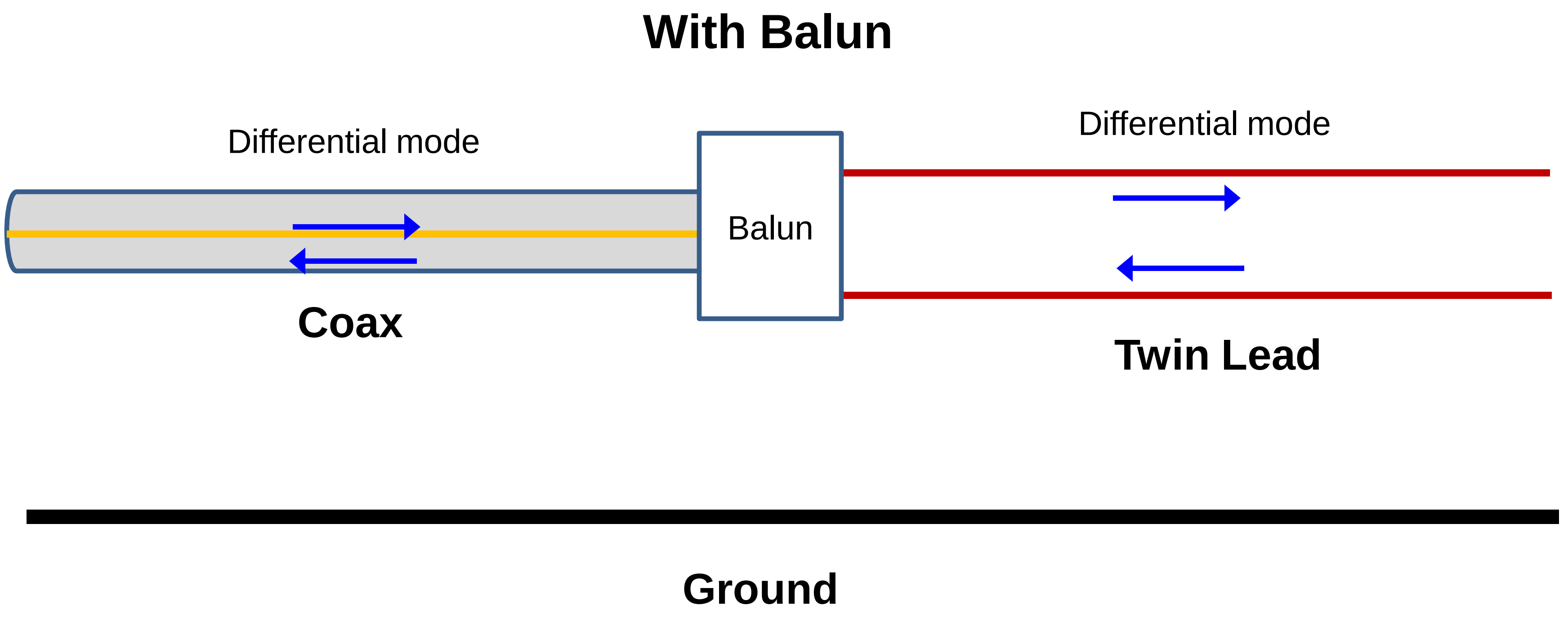
# Baluns (cont.)

Next, we use superposition.



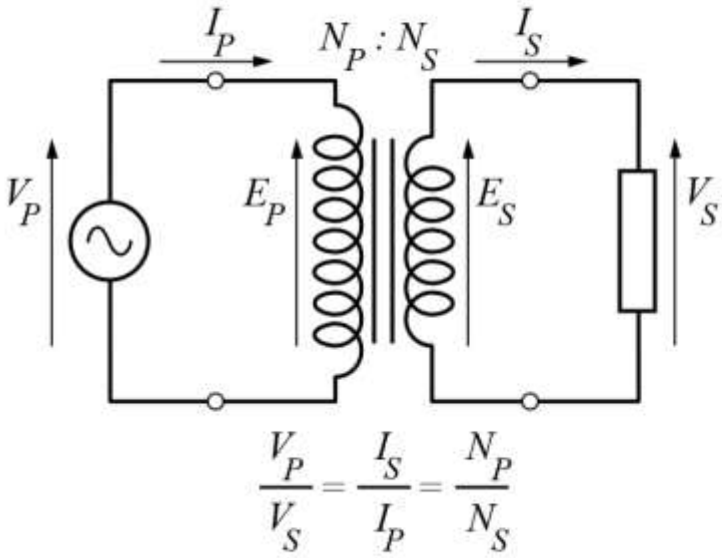
# Baluns (cont.)

A balun prevents common modes from being excited at the junction between a coax and a twin lead.

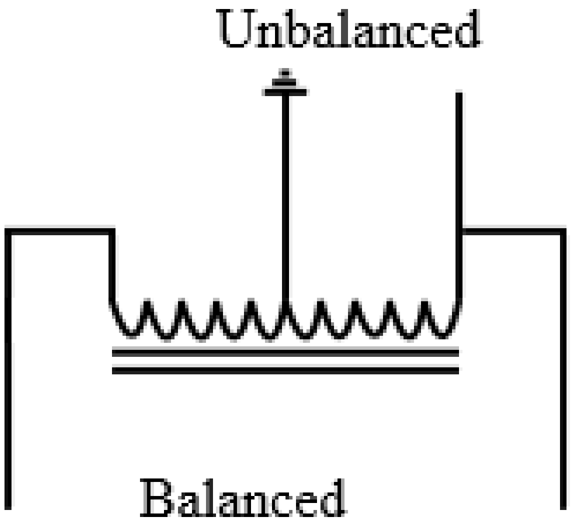


# Baluns (cont.)

One type of balun uses an *isolation transformer*.




The input and output are *isolated* from each other, so one side can be grounded while the other is not.



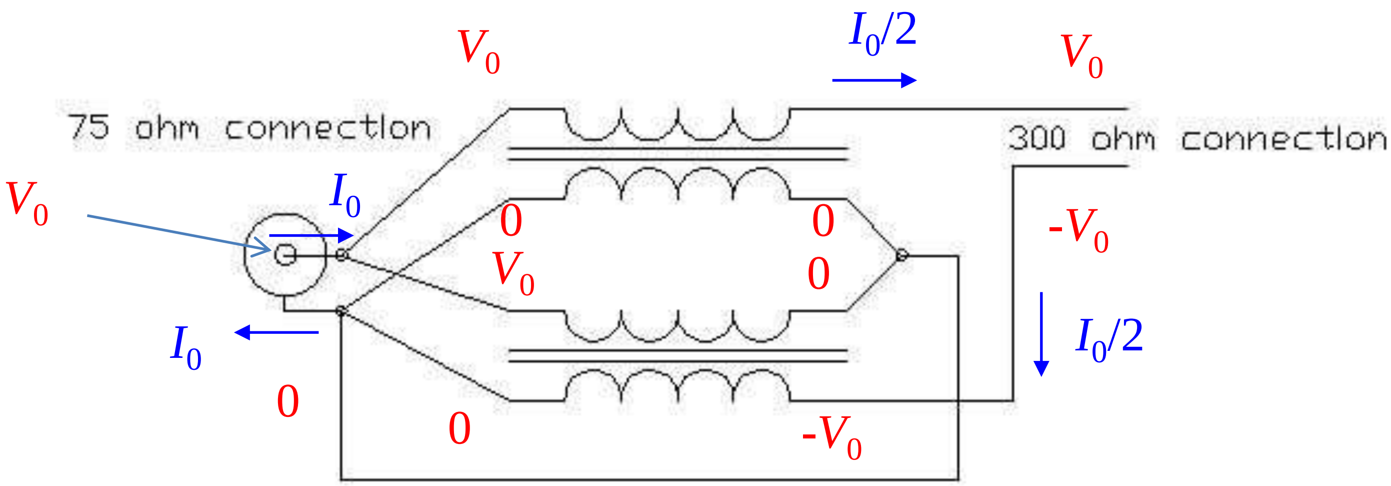

Circuit diagram of a 4:1 autotransformer balun, using three taps on a single winding, on a ferrite rod

https://en.wikipedia.org/wiki/Balun

# Baluns (cont.)

Here is a more exotic type of 4:1 impedance transforming balun.

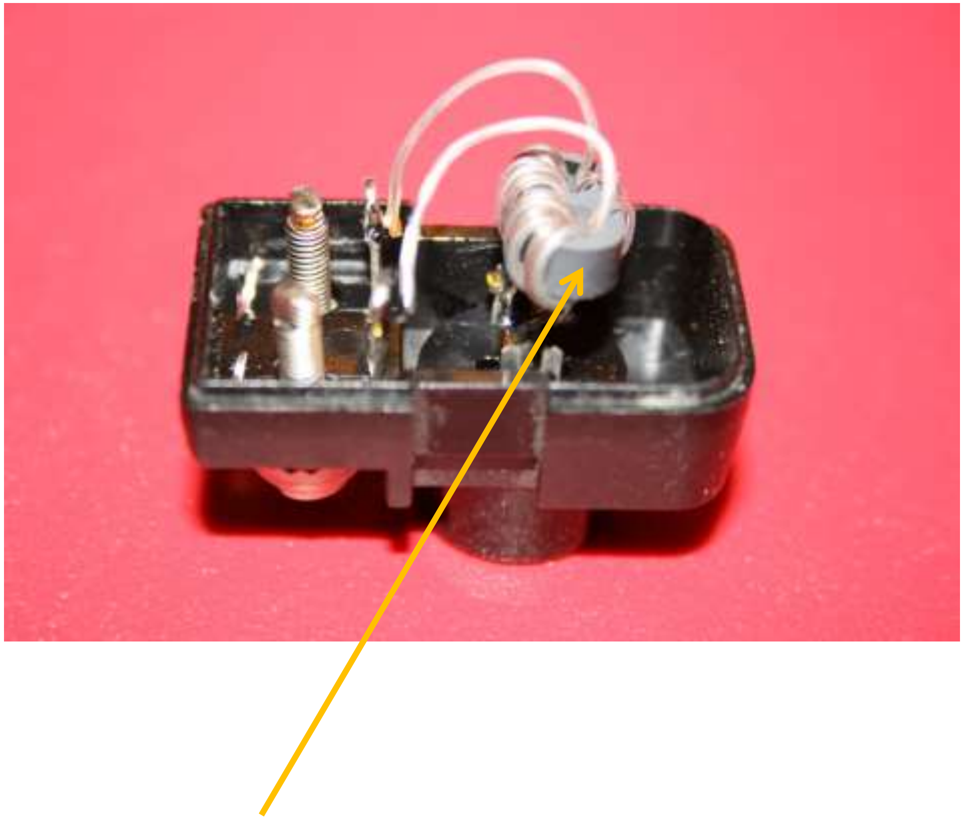
**“Guanella balun”**




This balun uses two 1:1 transformers to achieve symmetric output voltages.

# Baluns (cont.)

Inside of 4:1 impedance transforming VHF/UHF balun for TV



Ferrite core

# Baluns (cont.)

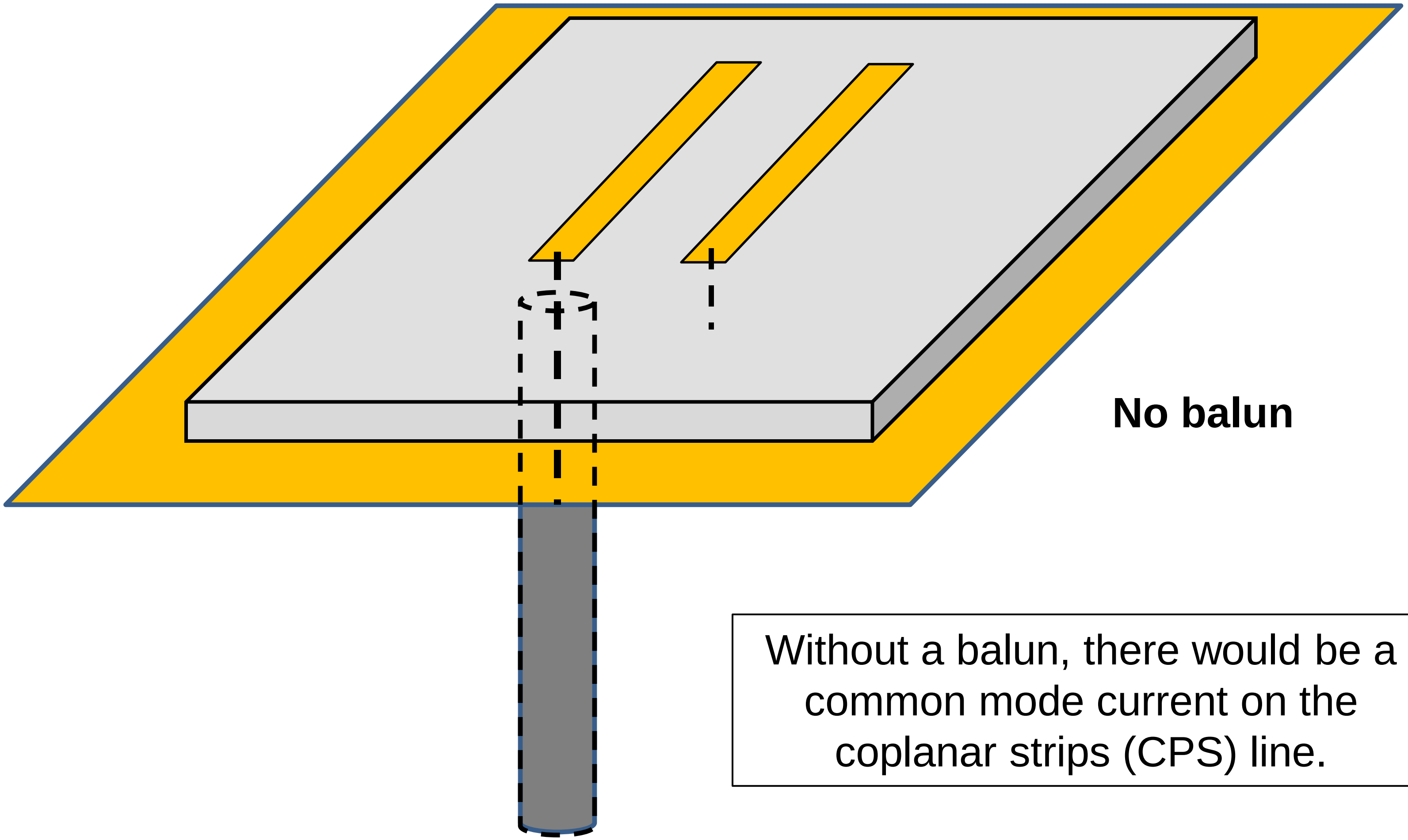
Another type of balun uses a choke to “choke off” the common mode.

A coax is wound around a ferrite core. This creates a large inductance for the common mode, while it does not affect the differential mode (whose fields are confined inside the coax).

https://en.wikipedia.org/wiki/Balun

# Baluns (cont.)

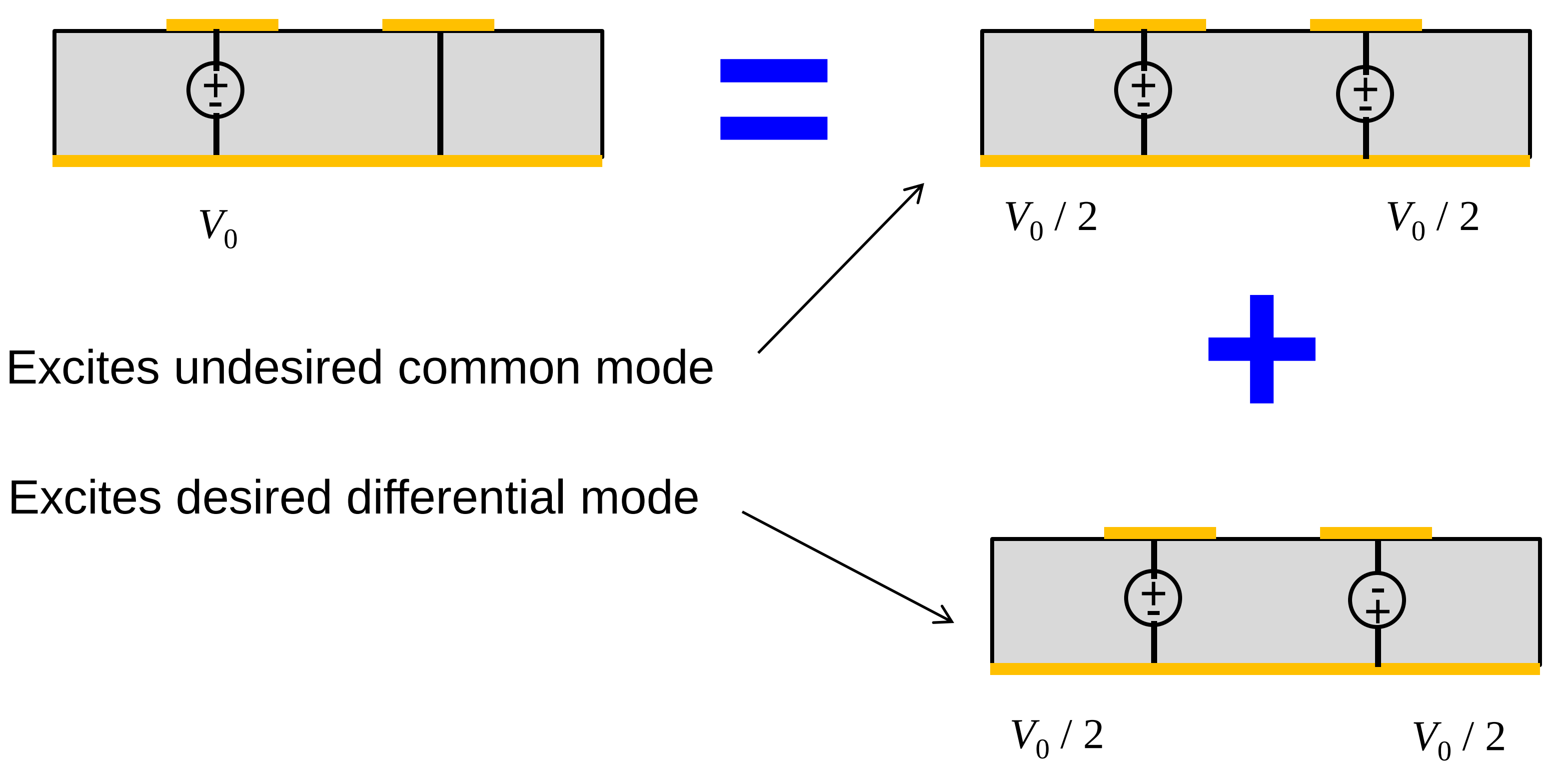
Baluns are very useful for feeding differential circuits with coax on PCBs.




Without a balun, there would be a common mode current on the coplanar strips (CPS) line.

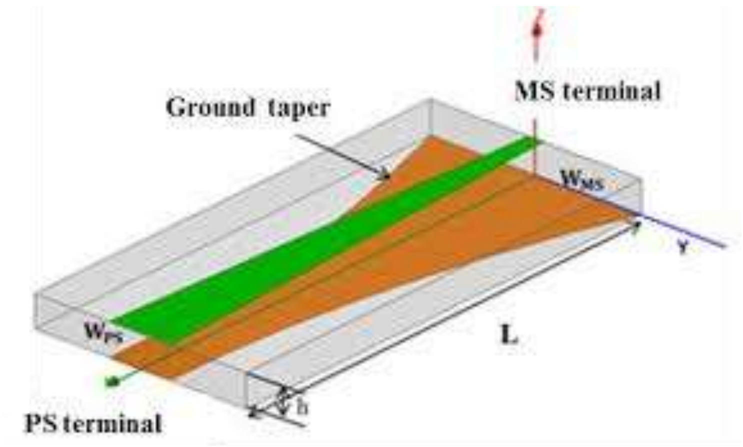
# Baluns (cont.)

Superposition allows us to see what the problem is.

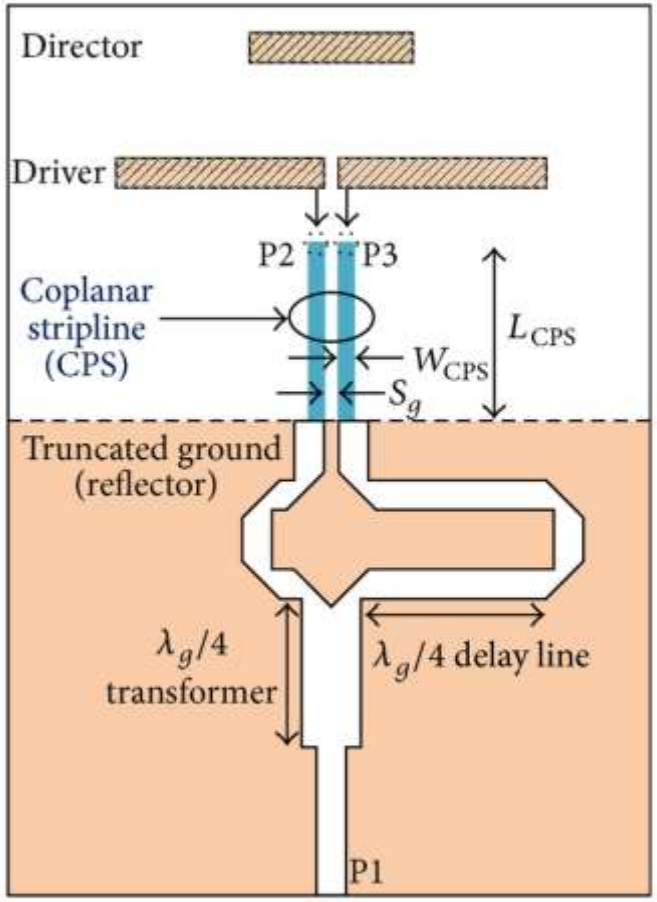


# Baluns (cont.)

Baluns can be implemented in microstrip form.



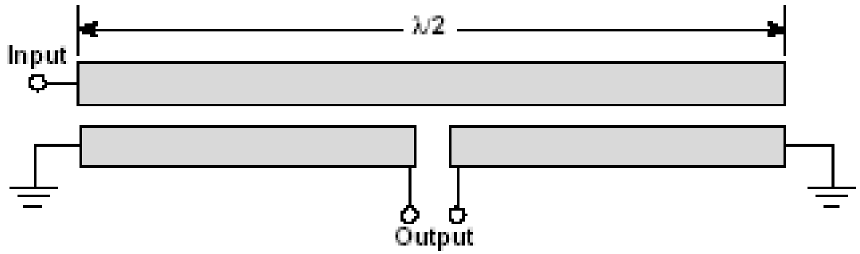

**Tapered balun**



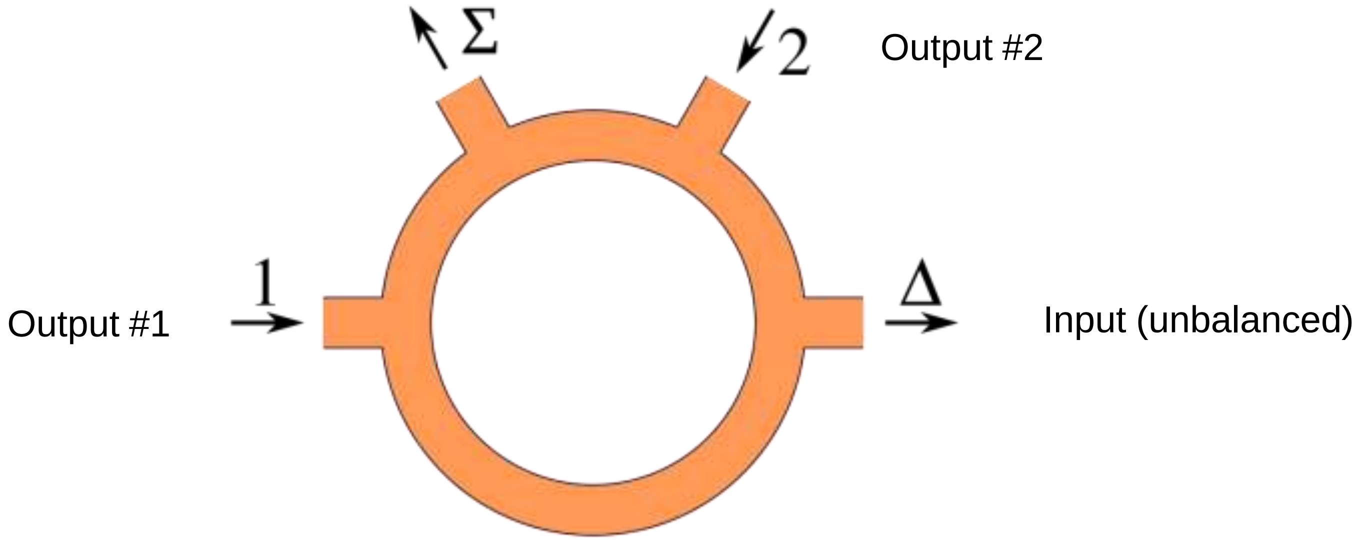

**Balun feeding a microstrip yagi antenna**

# Baluns (cont.)

Baluns can be implemented in microstrip form.



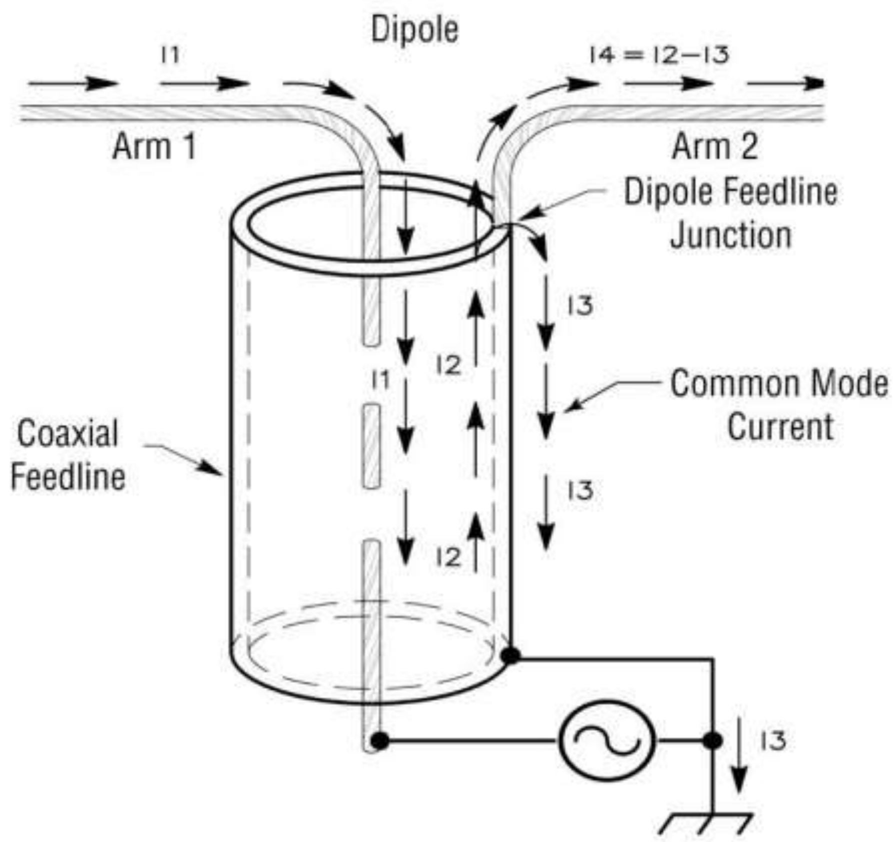

**Marchand balun**




**180º hybrid rat-race coupler used as a balun**

# Baluns (cont.)

If you try to feed a *dipole antenna* directly with coax, there will be a common mode current on the coax.




$I_1 = I_2$

$I_3 \neq 0$

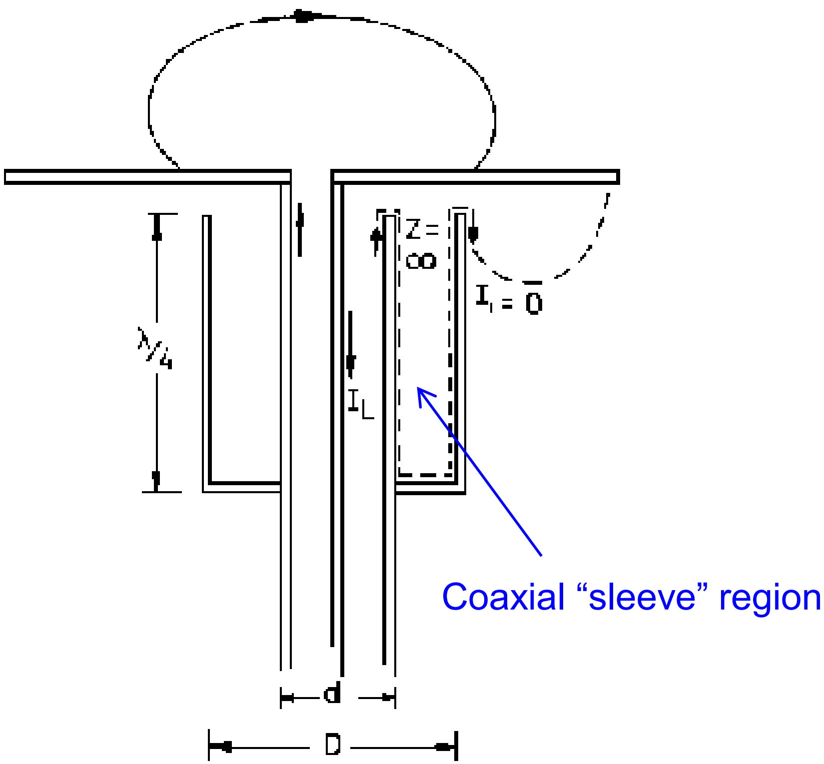
**No balun**

# Baluns (cont.)

Baluns are commonly used to feed dipole antennas.

A balun first converts the coax to a twin lead, and then the twin lead feeds the dipole.

**Coaxial “sleeve balun”**

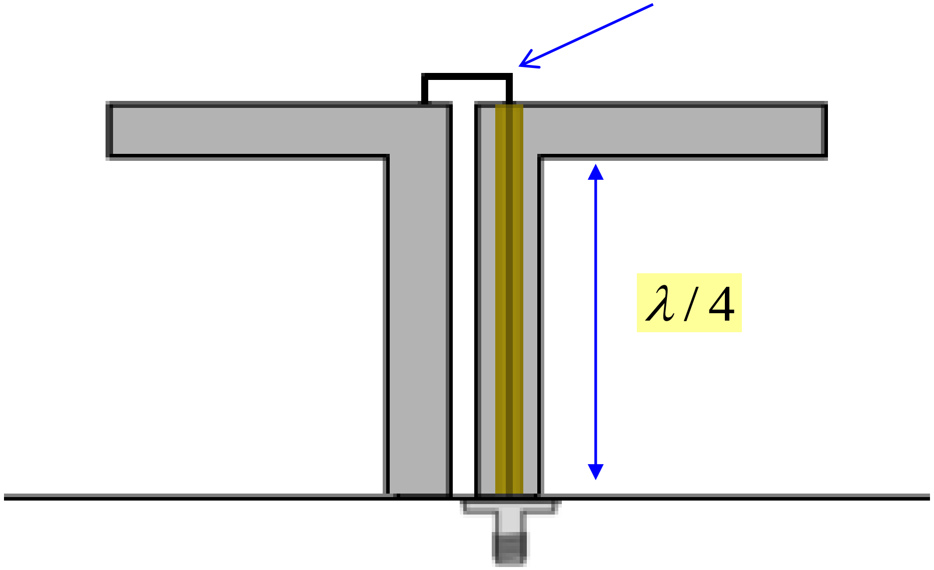



A “sleeve balun” directly connects a coax to a dipole.

# Baluns (cont.)

## Feeding a dipole antenna over a ground plane

This acts like voltage source for the dipole, which forces the differential mode.




The dipole is loaded by a short-circuited section of twin lead – an open circuit because of the $\lambda/4$ height.